THE UNIVERSITY OF ALBERTA

TWO-PHASE (AIR-WATER) SLUG FLOW IN HORIZONTAL AND INCLINED TUBES

by

© LEWIS R. VERMEULEN

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES

IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF SCIENCE

IN

CHEMICAL ENGINEERING

DEPARTMENT OF CHEMICAL AND PETROLEUM ENGINEERING

EDMONTON, ALBERTA

AUGUST, 1968

UNIVERSITY OF ALBERTA

FACULTY OF GRADUATE STUDIES

The undersigned hereby certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled: "Two-Phase (Air-Water) Slug Flow in Horizontal and Inclined Tubes" by L.R. Vermeulen in partial fulfilment of the requirements for the degree of Master of Science in Chemical Engineering.

ABSTRACT

The phenomenon of two-phase slug flow in the system air-water was studied in a one-half inch clear Lucite tube 40 feet in length. Pressure drop, pressure fluctuations, slug velocity, and slug frequency were measured over the 18.29 foot test section. These variables were studied in horizontal and inclined flow, at plus and minus seven degrees to the horizontal. The data have been cross-plotted to show the influence of the respective mass flow rates and tube orientation on an air-water system at 80 degrees Fahrenheit and essentially atmospheric pressure. Comparisons between the pressure drop correlations of Lockhart and Martinelli and of Kordyban were made with the data. The predictions were, however, found to be between 30 to 50 percent lower than the actual data. To improve on this, a semi-empirical theory based on a mechanistic flow model of slug flow was developed and compared with the data. The predictions by this model were found to be between 25 to 30 percent higher than the data over most of the range studied. By providing an upper bound to slug flow pressure drop, this model is superior to either of the above two models for slug flow.

ACKNOWLEDGEMENTS

The author wishes to express his sincere gratitude to Dr. J.T. Ryan for his guidance, encouragement, and his unfailing interest in supervising this investigation.

Acknowledgement is made to Mr. Geoff Walsh and his staff of the Chemical and Petroleum Engineering Shop for their assistance in the design and construction of some sections of the equipment.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF FIGURES (continued)

CHAPTER I

INTRODUCTION

The case of single phase flow in pipes has been satisfactorily developed and it is conceded that the flow of a Newtonian fluid offers a fairly simple problem. However, the introduction of another phase makes the task considerably more complex. It might seem that the addition of a small amount of liquid to a gas stream would still allow the pressure drop to be calculated by a gas flow equation. However, if the calculation were made on this basis, the result might be in error by as much as five hundred percent. Errors of this order of magnitude indicate that there is little understanding of the basic mechanism of two-phase flow.

As a consequence of this lack of knowledge many designers prefer to avoid two-phase flow wherever possible. There are circumstances, however, where some technical or economic advantage can be gained by utilizing the characteristics of two-phase flow, such as in heat transfer, mass transfer, agitation, or transportation of multiple phases, as could be encountered in wet gas gathering systems. Consequently some predictability of the fluid behavior must be available to the designer.

In general, a two-phase mixture separates when flowing into a pipe. The gas flows away rapidly and leaves the liquid to move along at a much slower velocity. However, when the fluids encounter obstructions, such as bends, valves, or a change in slope, the liquid may bridge the tube cross section, resulting in the formation of slugs of liquid, separated by gas-filled voids. With changes in fluid properties,

such as those caused by the expanding gas, and changes in tube characteristics and orientation, the flow pattern may again resolve itself into any one of several distinct forms. The prediction of pressure drop and liquid inventory in the pipe is therefore very difficult. For example, Flanigan (21) reported that the pressure drop in a pipeline had been predicted to be 30 psig. This soon built up to 300 psig!

It is therefore evident that the behavior of two-phase flow cannot be predicted by the utilization of the existing single phase flow equations. The fluid mixture cannot be simply described as being laminar, transitional, or turbulent. In single phase flow, the frictional pressure drop is inversely proportional to the fifth power of the tube diameter. In multiphase flow, however, this diameter can be effectively reduced for each phase, resulting in a drastic increase in pressure drop. While single phase momentum transfer occurs by either a molecular or turbulent mechanism, two-phase flow has an additional bulk momentum transfer, as well as an acceleration term.

It is remarkable that a general, but empirically derived, correlation as that of Lockhart and Martinelli (30), which was developed without regard for flow patterns, can be applied over practically the whole range of variables. But neither this, nor other correlations developed in a similar manner, or from a separate treatment of each phase, adequately describe two-phase flow. If one knows the flow pattern and all the physical and geometric properties of the system, the best correlations will predict the pressure drop with an accuracy of about 25 percent (2,3). To overcome this problem the general direction of research has, therefore, been directed, in recent years, towards obtaining analytical solutions for specific flow geometries.

The most frequently cited reference for flow patterns is Alves (1). He categorized the patterns into seven regions based on visual observations of flow in horizontal pipes. In order of their appearance with increasing gas rate and at a constant liquid rate they are illustrated in Figure 1 and are called:

1. bubble flow
2. plug flow
3. stratified flow
4. wavy flow
5. slug flow
6. annular flow
7. dispersed or spray flow.

The majority of these flow forms have been studied rather extensively, with the exception of horizontal slug flow. It is perhaps the most troublesome form of flow to be encountered in the design and operation of multiphase pipelines.

Slug flow is formed when the liquid bridges the tube cross section and is therefore characterized by a pattern in which all or part of the cross section of the tube becomes alternately filled with liquid plugs, separated by gas-filled voids. These slugs can travel at a considerably higher velocity than would be expected from the overall volumetric rate. When vented to the atmosphere, slug flow is characterized by violent pressure fluctuations which are propagated through the pipeline. If these pulsations are not damped, they could cause severe damage to the receiving vessels at the end of the line.

To reduce the effects of slug flow, or to completely avoid this troublesome pattern it is therefore necessary to study the problem in detail.

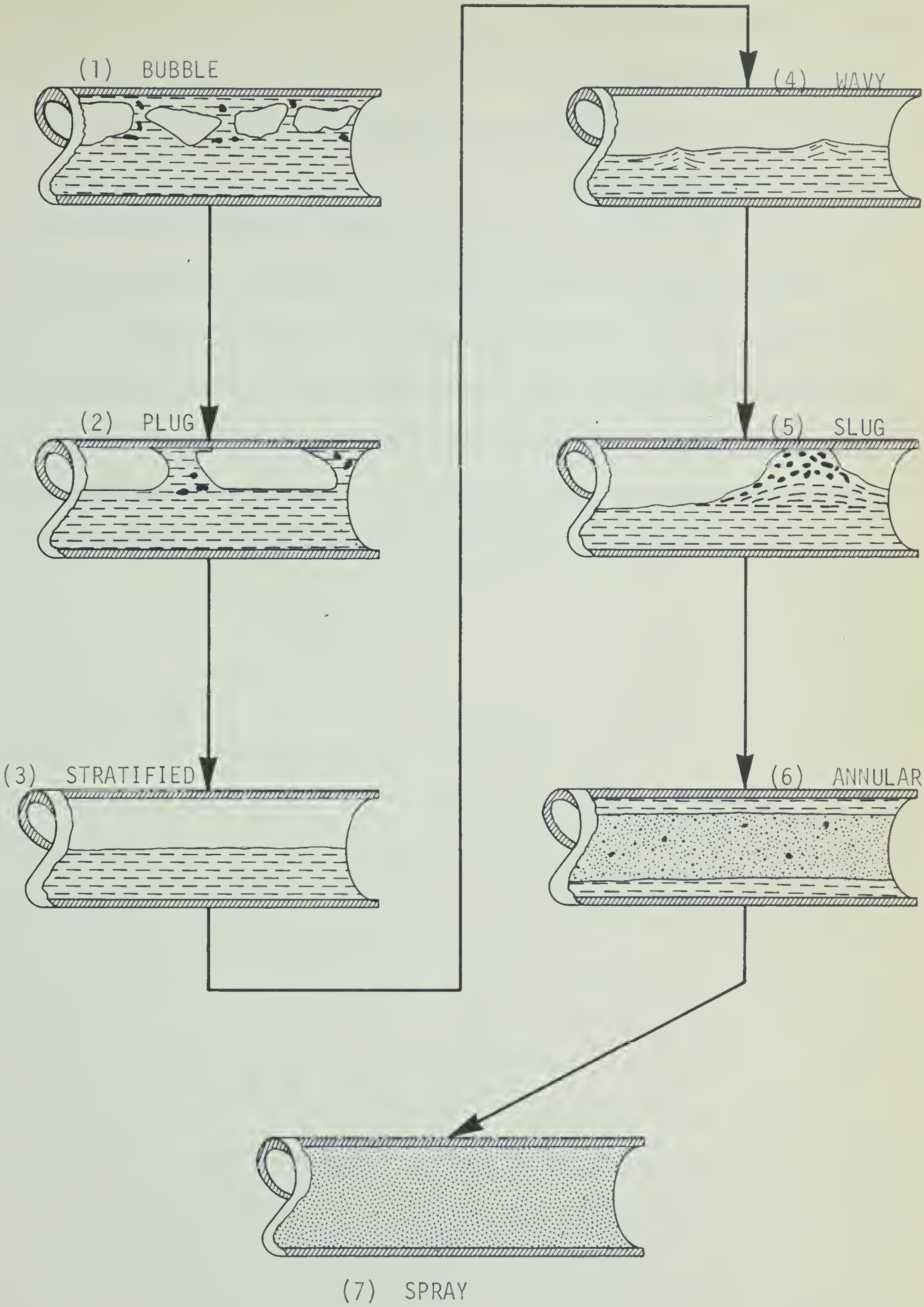


FIG. 1 - HORIZONTAL FLOW PATTERNS AFTER ALVES

It was not until 1961 that this complex flow pattern was first analyzed. Kordyban's (27) mechanistic flow model for slug flow was used to correlate his pressure drop data. In 1963 he (28) experimentally analyzed the behavior of slug flow by measuring slug velocities and frequencies, pressure drops and fluctuations, and velocity distributions. The nature of this work was largely exploratory since it was carried out in the slow slug flow region. The work was also left incomplete.

This thesis could, perhaps, be considered to be an extension of Kordyban's work. Similar measurements were made, but in the region of highly slugging flow. In addition, slug flow in pipe-lines was simulated and studied by inclining the tube to positive and negative angles of seven degrees (20) from the horizontal.

CHAPTER II

LITERATURE REVIEW

Two-phase flow has been studied periodically for the past sixty years, but it was not until the early 1940's that this phenomenon was begun to be analyzed in detail.

In the field of gas-liquid flow the literature is quite extensive. A recent annotated bibliography (26) cites over 2800 references for the period 1950 to 1962. Gouse (23) has compiled a general index of 5253 references and a further review (12) from 1931 to 1962 lists 308.

Initial attempts to describe the behavior of a gas-liquid mixture in terms of single phase flow have proved to be largely unsuccessful. Frictional pressure drop has been calculated from mixture fluid properties such as density and viscosity, but acceleration effects due to the expanding gas cannot be calculated analogously. The artificiality of the assumption of homogeneous flow, and particularly of the assumption that the gas and liquid velocities are the same has been realized for some time. The relative movement, or slip, between the phases has, therefore, been studied extensively. This phenomenon accounts for the ratio of liquid and gas in the pipe, or holdup, which is not only dependent upon the relative rates of the phases but also on tube characteristics, orientation and fluid properties.

To fully understand the basic mechanisms of two-phase flow it is therefore necessary to study these aspects and their interrelation in detail. While the earliest work was primarily concerned with the study and prediction of pressure drop, increasing emphasis has been

placed on analysis of the individual flow patterns encountered in gas-liquid flow. This sequence, however, has not always been followed. Consequently this review of the literature has been divided into three sections: pressure drop, flow patterns, and holdup.

2.1 Pressure Drop

The earliest two-phase flow literature was primarily concerned with the theory and application of air-lift pumps. A discussion of these reports by Wood (44) points out that a simple mathematical theory on the action of the airlift pump was presented by Lorenz (31) in 1909. Davis and Weidner (16) in 1911 conducted tests with vertical air-water flow and determined the efficiency of actual work done to that obtainable from theory.

In 1930 Uren et al (42) reported on the behavior of air-oil mixtures. He derived a parameter similar to a Reynolds number, using a specific gravity based on the volume fractions of the air and oil, and an absolute viscosity calculated from Poiseuille's formula. A plot of Uren's parameter versus the Fanning friction factor, on logarithmic coordinates, resulted in a linear relationship for all values of this Reynolds number. It was realized, however, that since slippage and non-homogeneity of the air-oil mixture had not been considered in this relationship, it was open to criticism.

A study by Moore and Wilde (35) in 1931 took slippage into account. A comparison of the actual friction drop with that predicted by the Fanning equation by subtracting the hydrostatic and velocity heads from the total pressure drop showed large deviations. This procedure was therefore considered inapplicable for the flow of hetero-

geneous fluids. Nevertheless their empirical expression correlated their data, but was found to be unsatisfactory for longer test sections such as in gas-lift oil wells.

The analysis of data from 28 producing wells, carried out by Nowels (37) in 1932, resulted in a "turbulence factor" analogous to Reynolds number. The parameters were calculated using an average density based on top and bottomhole pressure and an average velocity calculated from the volumetric rates of the oil and gas under average pressure. The absolute viscosity was calculated from the linear relationship of oil-gas mixture viscosities and fraction oil in the mixture at atmospheric conditions. Again, as in the work of Uren et al (42), a linear relationship was obtained in the logarithmic plot of "turbulence factor" versus friction factor. The result was that, for a given value of "turbulence factor", the friction factor increased with the diameter, which is contradictory to the effect in the horizontal flow phenomenon. Pressure gradient was greatest for pipes of small diameter.

Up to this time the great majority of publications was concerned with vertical two-phase flow. In 1939, Boelter and Kepner (6) published the results of their investigation into horizontal, as well as inclined flow. Air-oil and air-water mixtures in one-half and three-quarter inch standard pipes were studied in the horizontal position and at an upward slope of 1:6. The flow was observed through glass sections and a variety of flow patterns were described, including slug flow. In computing the "power" [sic] to lift oil up a sloped pipe it was assumed that the oil flowed quiescently in the lower portion of the pipe. The "equilibrium quantity" [sic] of oil within the pipe was determined as a function of air-oil ratio, air velocity, and pipe diameter.

The authors set up the equipment such that a horizontal section of pipe preceded the inclined one. Under these conditions the pressure gradient in the sloped line was observed to increase. In addition the level of oil on this section increased, thereby causing the oil level and pressure gradient in the horizontal section to increase as well. The pressure drop in the sloped line was found to be almost doubled at high oil rates and 10 cfm air rate, while the increase in the horizontal section was about twenty-five percent.

In 1944, Martinelli and co-workers (34) published the results of a study of isothermal two-phase, two-component flow of air and eight liquids, which included benzene, water, and S.A.E. 40 oil. The results of the tests which were conducted in a one inch glass pipe and a one-half inch galvanized pipe caused the authors to conclude that tube diameter, liquid and gas rates, and liquid and gas viscosities and densities and surface tension affected pressure drop. Liquid viscosity had the greatest influence on pressure drop if the liquid was flowing "viscously" [sic], while surface tension influenced the flow patterns appreciably.

In developing their general pressure drop correlation, Martinelli et al (34) assumed that the two-phase pressure drop is a function of single phase pressure drop if either fluid were flowing alone. Based on this postulate, these four possibilities could occur:

1. gas turbulent - liquid turbulent
2. gas turbulent - liquid viscous
3. gas viscous - liquid turbulent
4. gas viscous - liquid viscous

The first three mechanisms were analyzed, although only the

first two could be observed. The fourth, it was stated, probably could not occur; however, Martinelli could not have considered very slow stratified flow, where laminar flow for each phase is a possibility. In the development of their correlations, Martinelli et al further postulated that

1. The flow pattern is homogeneous along the length of the tube. This implied that such patterns as slug and plug flow were eliminated from consideration.

2. Pressure drops in each phase are equal.

In 1949 Lockhart and Martinelli (30), using additional data with tubes from 0.0586 to 1.017 inches inside diameter and various liquids, improved on the correlations of Martinelli et al (34). A new general parameter applicable to all four types of flow was proposed. Furthermore, slippage between the phases, determined by the fraction of the tube filled with each phase, i.e., holdup, was incorporated.

Alves (1), in 1954, issued his report on the study of two-phase flow for the design of pipe-line contactors. While his work was primiarly concerned with the investigation of the various types of flow patterns to be found in two-phase flow, he also reported pressure drop data. His slug flow pressure drop data were always higher than the predictions by the Lockhart-Martinelli correlation.

In 1955 Chenoweth and Martin (11) proposed an improved correlation for high pressure, turbulent two-phase flow in horizontal pipes of large diameter. This development was precipitated by the inapplicability of the Lockhart-Martinelli (30) correlation to predict

pressure drop for pressures above 50 psig and for large diameter tubes.

The test sizes were 1.5 and 3.0 inches tube diameter. The correlation incorporated a parameter which was equal to the ratio of the fictitious all-gas to all-liquid system. Furthermore, any fittings such as elbows, valves, and orifices as may be found in the test section, were included.

A comparison of the data with the Lockhart-Martinelli correlation showed best agreement at atmospheric pressure. The largest deviations were found to occur at 100 psia in the 3 inch pipe, where the predicted pressure drop was higher than the observed by a factor ranging from 1.4 to 2.5. Chenoweth and Martin's own correlation, on the other hand, represented all of the data within plus and minus 50 percent, with 92 percent of the data falling within 35 percent. The correlation was also found to fit the data of other investigators, including those of Alves (1).

In 1957, Hoogendoorn (24) reported his results on air-water and air-oil systems in horizontal smooth pipes with inner diameters ranging from about 1.0 to 5.0 inches. He stated that the Lockhart-Martinelli (30) correlation was not valid for plug, slug, and froth flow if the gas density differed from that of air at atmospheric pressure. For wave, mist, and annular flow it was inadequate under any conditions.

Hoogendoorn attempted to overcome these deficiences by developing a new correlation. This was successful only for the lower range of gas superficial velocity. The standard deviation between his data and the correlation was 6 percent. In the higher ranges, where slug flow prevails, the standard deviation rose to 14 percent. By

comparison with Lockhart Martinelli's correlation, the standard deviation from his data was 13 percent.

The same year Brigham et al (7) studied gas-liquid flow in a 1.975 inch pipe, horizontal and inclined at 12.4 degrees. The fluids used were air and No. 10 S.A.E. oil and air and water. The data were compared with the correlation of White (43). For the regions of wave, cresting, and semi-annular flow, the predicted values showed good agreement with the data. In slug flow, however, the measured pressure drop was much greater than predicted. It was also found that for nominally stratified flows the angle of slope did not affect the overall pressure drop. In a comparison of the inclined slug flow data with the predictions of the Lockhart-Martinelli (30) equation, the predicted values were five to six times smaller than observed.

With the data he collected from multiphase pipeline field tests, Flanigan (21) in 1958, modified existing design equations to improve the predictability of frictional and static head pressure drop. His analysis showed that

1. practically all of the pressure drop occurred in the uphill sections of the line and,

2. the pressure drop in the line decreased as the gas rate was increased.

The latter observation is contrary to that observed in horizontal flow, but has been shown to be true in vertical lines (10, 22, 39).

The pressure drop in the pipeline was obtained by applying a pipeline efficiency, which is strongly dependent on flow patterns, to the frictional pressure drop. The static head was also corrected, since the two-phase pipe is never completely filled with liquid. Com-

parison of predicted with actual pressure drop was found to result in a maximum deviation of ± 14 percent, with the majority falling within ± 5 percent.

One of the first mechanistic flow models for slug flow was presented by Kordyban (27) in 1961. The resulting equation resembles rather closely the applicable portion of the empirical Chenoweth-Martin (11) correlation. The two-phase pressure drop was calculated, in each case, by dividing the single phase pressure drop by the liquid volume fraction raised to a power between 0.75 and 1.00. Kordyban's correlation was developed by considering all the liquid to flow in slugs, separated by gas-filled spaces, with the gas pressure drop assumed to be negligible. Only frictional pressure drop was considered. However, he realized that in the steam-water system the pressure drop produced by acceleration of liquid due to increasing amounts of vapor could be calculated to be over 50 percent of the total pressure drop. For the 0.420 inch tube agreement of the prediction with his data was quite good; however, for the 0.315 inch tube the calculated values were found to be 20 to 30 percent lower than the experimental data.

In 1963 Kordyban and Ranov (28) issued a further report on slug flow in horizontal tubes. Pressure drop in the 1.25 inch plexiglas tube was measured with differential pressure transducers rather than with U-tube manometers, as had been the usual practice up to this time. This enabled them to measure not only average values, but also the pressure fluctuations which occur when slugging flow is vented to the atmosphere. At the time of its presentation the work was incomplete, thus, no new correlation was proposed. The approach in this study of slug flow was to measure the characteristic behavior

of the slug mechanism. Variables such as slug velocity and frequency, pressure drop and fluctuations, and velocity distribution were therefore measured to enable the authors to improve on the previous correlation of Kordyban (27). This earlier mathematical model, the authors stated, "did not describe satisfactorily the behavior of slug flow, because it had been based on insufficient data".

In 1964 a critical review of the then known correlations for two-phase flow was reported by Dukler and Wicks (17). A search of the literature revealed over 20,000 individual measurements of pressure drop. By statistically culling the available data, 2620 points remained. An overall comparison of five correlations with the data showed the Lockhart-Martinelli (30) correlation to be clearly superior to those of Baker (4), Bankoff (5), Chenoweth-Martin (11), and Yagi (45).

Dukler et al (18) further investigated frictional pressure drop through similarity analysis. The basic equation was solved for four sets of boundary conditions. Comparison of these results to the Lockhart-Martinelli (30) correlation showed them, in general, to be superior to the latter, while the opposite was true at low viscosities with small diameter tubes.

Hughmark (25) in 1965 presented a correlation which utilized the amount of liquid in situ, i.e., holdup, in horizontal slug flow. This holdup was estimated from the relationship of bubble velocity and liquid slug Reynolds number. Comparison with the results of several other investigators, including Alves (1), Baker (4), and Brigham (7) was inconclusive. The authors stated that this occurred "probably because some of the latter data were not for slug flow."

Chisholm (13), in 1966, presented a note on the relationships

between friction and liquid cross section during two-phase horizontal flow. He correlated data to within 20 percent in smooth and galvanized tubes.

2.2 Flow Patterns

The first reported flow pattern observations were those of Davis and Weidner (16) in 1911. They found that with an increasing air rate at a constant water rate the flow pattern in a vertical tube would change from one of small bubbles in the water to long bubbles or pistons, which disappeared when the air to water ratio became large.

Boelter and Kepner (6), who in 1939 carried out studies of both horizontal and inclined flow, installed glass sections into their 1/2 inch and 3/4 inch tubes for flow pattern observations. Various patterns were noted, among them slug flow. The difference in behavior of oil and water under the influence of a high air rate was also described. In the transition from wavy flow, the wave crests of oil flow tended to break up into spray, while water formed a slug flow pattern.

A year later, in 1940, Cromer and Huntington (15) conducted extensive investigations using 2-inch standard pipe fitted with observation windows. Patterns of bubble, slug, froth, annular and dispersed flow were observed with increasing flow rate of air.

In 1944, Martinelli et al (34), gave a detailed description of the change in flow patterns with a decrease in water rate and a constant gas rate. Their conclusions were that "the microscopic behavior of the two-phase flow system is extremely complex. Since, however, the static pressure drops from the water and water with surface

tension reducing additive were almost identical, although the detailed flow patterns were quite different, it appears that it should be possible to analyze the problem from the macroscopic point of view and obtain results which are of utility from an engineering standpoint, even though the details of flow mechanism are not clarified." This conclusion, the authors felt, was justified in the same manner as the employment of a friction factor for the prediction of pressure drop in turbulent flow, without the need for consideration of the complex details of turbulence. As a result a two-phase flow system was considered to contain the two fluids as a homogeneous mixture.

Lockhart and Martinelli (30) reconfirmed these conclusions in 1949. Only homogeneous flow, i.e., complete mixing of the phases, was considered in their correlations.

In the discussion of this paper (30) Carl Gazeley, Jr. and O.P. Bergelin (University of Delaware at Newark, Delaware) concluded, however, that a completely successful correlation for two-phase flow could not be obtained until the transition points between the various types of flow had been determined.

The same year Kosterin (29) published his results on the influence of tube diameter and position upon hydraulic resistance and flow structure of gas-liquid mixtures. Visual observations, still photographs, and motion pictures of gas-liquid flow structures in tubes of 1, 2, 3, and 4 inches were used to construct charts showing the distribution of the various regions. Slug flow was described as occurring in the region between divided flow and strongly dispersed flow, where the gas or vapor moved in the form of large bubbles, which occupied a considerable part of the diameter.

The possible flow configurations were divided into five distinct regions.

1. Divided flow: with gas in the upper part and liquid in the lower part of the tube cross section.

2. Quiet plug flow: with large gas-bubble plugs, with a clean boundary surface and without froth formation.

3. Plug flow: with froth formation in the trailing part of the bubble.

4. Plug flow: with froth formation over the entire gas-liquid boundary surface along the gas plugs.

5. Strongly dispersed flow, in which the gas-liquid mixture flows in the form of a more or less uniform froth: a gas-liquid emulsion.

Flow condition charts were constructed with percent gas as the ordinate and mixture velocity, based on the gravimetric delivery rate of the mixture, as the abscissa.

In inclined flow, at 70 degrees to the horizontal, it was noted that stratified flow did not occur and thus, it was stated, was a phenomenon particular to horizontal flow, only.

In 1954, Alves (1), in carrying out his work on cocurrent pipeline contactors, gave an even more detailed description of the various possible flow configurations. He classified horizontal two-phase flow into seven more or less distinctive regions (Figure 1). With an increasing gas rate and at a constant liquid rate, they are (8):

1. Bubble flow: Bubbles of gas move along the upper part of the pipe at approximately the same velocity as the liquid.

2. Plug flow: Alternate plugs of liquid and gas move along the

upper part of the pipe.

3. Stratified flow: Liquid flows along the bottom of the pipe and gas flows above, over a smooth liquid-gas interface.

4. Wavy flow: Similar to stratified flow except that the gas moves at a higher velocity and the interface is disturbed by waves travelling in the direction of flow.

5. Slug flow: Waves are picked up by the more rapidly moving gas to form a slug, which passes through the pipe at a velocity greater than the average liquid rate.

6. Annular flow: The liquid flows in a thin film around the inside wall of the pipe and the gas flows at a high velocity as a central core.

7. Dispersed or spray flow: Most of the liquid is entrained as a spray by the gas.

A plot depicting these regions of two-phase flow, shown in Figure 2, was set up by Baker (4) in the same year. Valid flow pattern predictions can frequently be made by this plot, but more often, only an indication of the possible flow regime can be expected for a given set of conditions.

In 1957, Hoogendoorn (24), constructed flow pattern charts using the parameters of Kosterin (29). The results were similar, but because the only variables taken into account were mass velocity and percent gas, the charts, in each case, were particular only to the specific system under investigation.

Several such area plots have been constructed in the course of investigations into two-phase flow. However, researchers in the field generally agree (8) that, to date, there is no method in the

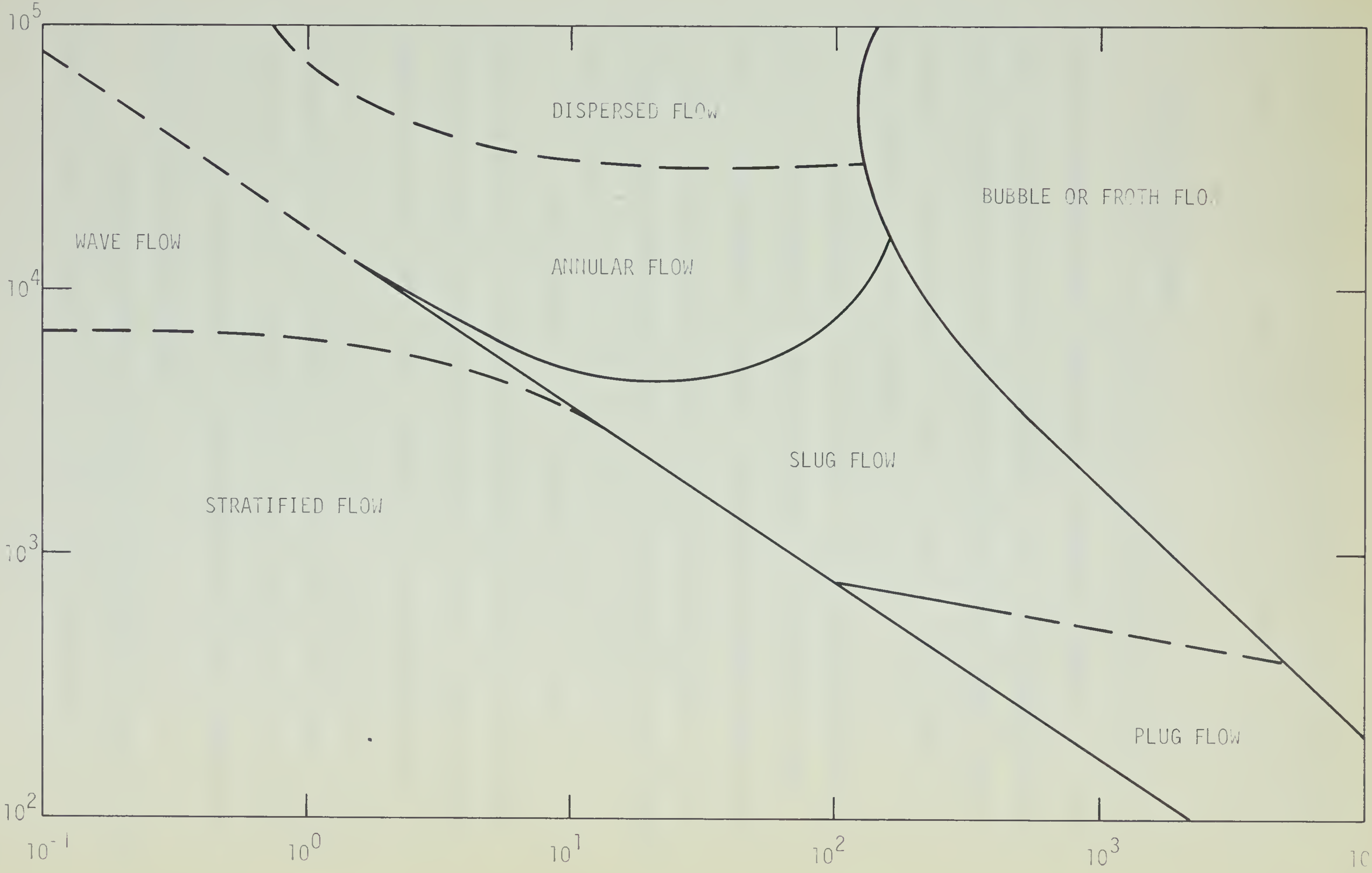


FIG. 2 - BAKER'S CHART FOR TWO-PHASE FLOW REGIONS

published literature which adequately describes or establishes just what type of flow will occur under the range of possible conditions.

2.3 Holdup and Slippage

The apparent slippage of the gas through the liquid was realized by the earliest investigators, including Lorenz (31) in the study of vertical two-phase flow systems. The first reported measurements of holdup were those of Moore and Wilde (35) in 1931. Quick closing valves were employed to trap the mixture. The authors reported that holdup was independent of viscosity.

For horizontal flow, the first reported measurements and correlation of holdup were those of Lockhart and Martinelli (30). The liquid was trapped in the tube, then washed out with large amounts of volatile solvent, and the excess solvent was evaporated. A plot of holdup versus the square root of the ratio of liquid to gas pressure drop resulted, for all practical purposes, in a single curve.

In 1957 Hoogendoorn (24) measured holdup by means of a capacitive probe. His results were checked by adding an oil-soluble radioactive tracer to measure the radiation intensity, which depends linearly on liquid holdup. In a comparison of Lockhart-Martinelli's (30) correlation with these data, Hoogendoorn reported deviations, but their severity was not disclosed. He correlated his own data with the type of slip velocity relationship normally employed in vertical flow (35). No significant effect of pipe diameter or liquid viscosity was found in these velocity ranges.

The adverse effects of holdup in pipelines are clearly evident from the report of Flanigan (21) in 1957. As had been reported pre-

viously, the predicted pressure drop of 30 psig in a pipeline soon rose to 300 psig. This drastic increase was attributed to holdup occurring in the line.

In 1963 an improved method for holdup determination was used by Neal and Bankoff (36), who employed an electrical resistivity probe. The circuit consisted of a steel needle connected to a flip-flop circuit. The needle was traversed across the pipe diameter to measure bubble frequency at various positions. This procedure was found to be a great improvement on the earlier method in which the probe was stationary. Petrick (38) also used the traversing technique but found that it still only gave an average value.

Kordyban (28) in 1963, used a resistance probe to measure holdup. The apparatus utilized the variation of electrical resistance with liquid level between two parallel electrodes which extended almost completely across the tube diameter. Although slugs and waves could be detected, these results were affected by a lag in drainage of the liquid film from the probe.

In 1963 McManus (33) reviewed experimental techniques in two-phase flow. In the section on the measurement on density or void fractions, reference was made to the development of contact probes, conductance elements, radiation attenuation, and light absorption. His conclusion was that these methods will only give overall density or void fraction at pipe cross sections. Precise determination of mean film depth and surface character in stratified and annular flows are possible, however, non-homogeneous flows cannot be measured successfully.

Other investigators such as Hughmark (25) continued to employ methods of estimating holdup by means of ratio of the respective phase

volumetric rate to total volumetric rate, corrected with an experimentally determined factor.

The main purpose of the above methods of determining holdup was to develop correlations for predicting holdup and consequently pressure drop. Most of these methods are inadequate, as no completely reliable instrument for measuring holdup is available.

As a practical solution to the problem of holdup in multiphase pipelines, McDonald and Baker (32) developed an improved method of "pigging". Whereas pigging was traditionally used to remove hydrates, etc., this new method employs inflatable rubber spheres to remove the accumulated liquid. The use of these pigs, introduced into the line at predetermined intervals, was found to drastically increase pipeline capacity. Although this method is in wide scale use, it is economical only if the ratio of liquid to gas is relatively small. As the relative amount of liquid is increased, the authors stated, the advantages from sphere operation will decrease. Furthermore, the optimum interval between the introduction of pigs cannot be determined precisely and thus the rate of pigging is based largely on the predetermined maximum allowable pressure drop in the line.

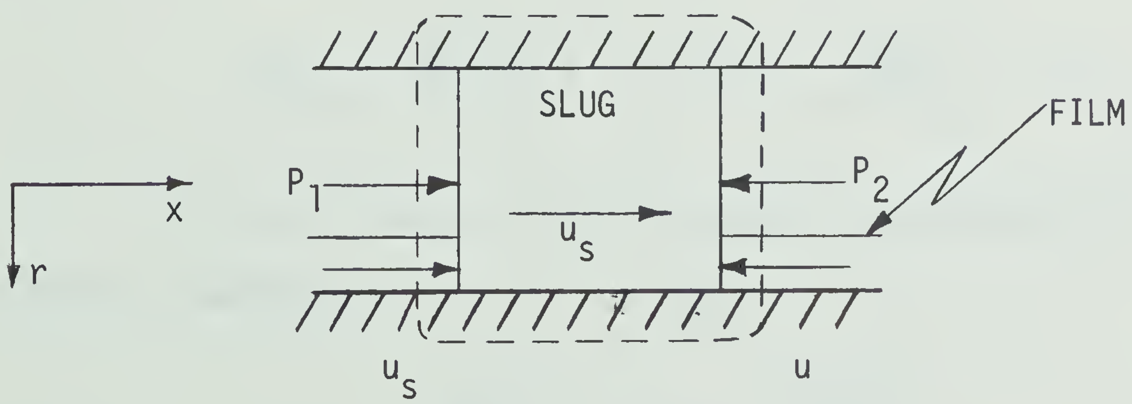
CHAPTER III

THEORY OF SLUG FLOW

Two of the existing two-phase flow correlations have been employed in this thesis as a comparison with the data. The correlation of Lockhart-Martinelli (30) was empirically derived for a homogeneous flow pattern, whereas that of Kordyban (26) was developed from a simplified model of slug flow. Although the first is widely used, a pattern such as slug flow was ruled out by the assumptions made in the derivation. Kordyban's model was found to be oversimplified since it was "based on insufficient data" (28).

Visual observations and the measurements of slug flow carried out by Kordyban in his later report (28) suggested the model shown below. It was observed that the liquid film at the front of a slug is picked up by the slug and its velocity is increased. A similar amount of liquid is dropped off at the rear of this slug and its momentum is gradually lost until the film is again picked up by the following slug.

3.1 Slug Flow Model



Consider a slug moving at a constant velocity u_s. The film, which is flowing at velocity u, is picked up by the slug and is dropped off at the rear. It is assumed that the film has been accelerated to the velocity of the slug, u_s. If it is further assumed that the slug does not accelerate, a momentum balance may be written around the control surface. This results in the following equation:

$$\Sigma F = 0 = \rho u_s^2 S - \rho \langle u^2 \rangle S + 2\pi R \int_0^{L_s} \tau_w \, dL_s - (P_1 A - P_2 A) \quad (1)$$

or

$$\underbrace{P_1 - P_2}_{\text{Pressure differential across one slug}} = \underbrace{\frac{\rho S}{A} (u_s^2 - \langle u^2 \rangle)}_{\text{Momentum exchange}} + \underbrace{\frac{2\pi R}{A} \int_0^{L_s} \tau_w \, dL_s}_{\text{Wall shear}} \quad (2)$$

where A = tube cross sectional area

S = film cross sectional area

Since the film is picked up by each slug in the tube, the total pressure drop is proportional to the number of slugs in the tube, N_s .

$$N_s = \frac{\text{slug frequency}}{\text{slug velocity}} \times \text{length of test section.}$$

Therefore

$$\Delta P_{TOT} = \frac{N_s \rho S}{A} (u_s^2 - \langle u^2 \rangle) + N_s \frac{2\pi R}{A} \int_0^{L_s} \tau_w \, dL_s \quad (3)$$

The shear force on the slug can be approximated by the Fanning Equation; therefore

$$\frac{2\pi R}{A} \int_0^{L_s} \tau_w \, dL_s = \frac{2 f \rho u_s^2 L_s}{g_c D} \quad (4)$$

Kordyban (27) regarded only this shear face term to be significant. He calculated the velocity of the slugs by assuming the liquid to flow alone in the tube.

To improve on this, the shear face term was evaluated in this thesis by assuming the gas to be incompressible. Thus the slug velocity equals the total fluid velocity,

$$u_s = \frac{Q_L + Q_G}{A}$$

Since the length of the slugs is not known, the fraction of the tube occupied by liquid slugs can be approximated as

$$N_s L_s = L \left(\frac{Q_L}{Q_L + Q_G}\right)$$

where L_s = length of slugs

L = length of test section.

Therefore equation (4) becomes

$$\frac{2\pi R\ N_s}{A} \int_0^{L_s} \tau_w dL_s = \frac{2f\rho u^2 L}{g_c D} \left(\frac{Q_L}{Q_L + Q_G}\right) \qquad (5)$$

To evaluate the momentum exchange term in equation (3),

$$\Delta P_m = \frac{N_s \rho S}{A} (u_s - \langle u^2 \rangle) \qquad (6)$$

several simplifying assumptions must be made. The film cross sectional area, S, may be calculated from the holdup, R_L, which was measured by Lockhart and Martinelli (30) by trapping the liquid in the tube. The liquid in the film is therefore equal to the total liquid less the liquid due to the flow rate.

Therefore

$$S = A(R_L - \frac{Q_L}{Q_L + Q_G}) = A\varepsilon$$

and equation (6) becomes,

$$\Delta P_m = N_s \, \rho\varepsilon(u_s^{\,2} - \langle u^2 \rangle) \qquad (7)$$

For simplicity, $\langle u^2 \rangle \simeq \langle u \rangle^2$. The resulting error of 16.5%, if the film is assumed to be in laminar flow, is justified in view of more inexact assumptions made above.

Evaluation of $\langle u \rangle$ is difficult since the film flows in the bottom of the tube. However, if it is assumed that the film flows on a flat plate, an approximation of $\langle u \rangle$ can be found by employing the Navier Stokes Equation.

$$\rho(\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z}) = - \frac{\partial P}{\partial x} + \mu(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2})$$

The equation of continuity is

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0$$

To solve these equations, several simplifying assumptions must be made. Since the flow is only in the x direction, u_y and u_z are zero. Therefore the continuity equation reduces to

$$\frac{\partial u_x}{\partial x} = 0$$

and therefore

$$\frac{\partial^2 u_x}{\partial x} \simeq 0$$

and there is no change in u_x with respect to z

$$\frac{\partial^2 u_x}{\partial z^2} = 0$$

The gas phase pressure gradient on the film in the x direction is negligible, therefore

$$\frac{\partial P}{\partial x} = 0$$

The equation is therefore reduced to

$$\frac{\partial u_x}{\partial t} = \nu \frac{\partial^2 u_x}{\partial y^2}$$

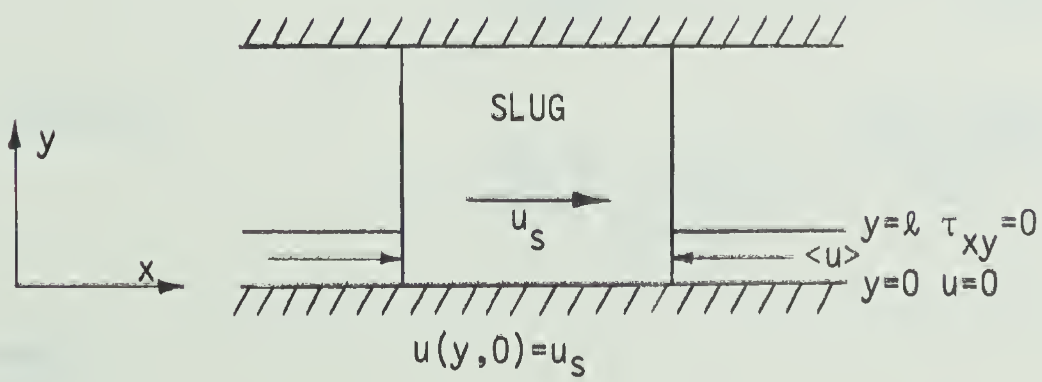
which must be solved for u_x.

This form, however, is similar to the heat transfer equation for temperature in a slab.

$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2}$$

A solution to the film slowdown problem may therefore be found by considering the momentum loss to occur analogously to heat loss.

Again considering the diagram,



the velocity of the film leaving the slug is equal to the slug velocity u_s. The film then begins to slow down since the velocity at the wall is equal to zero. To determine the average film velocity in the x direction, <u>, it is necessary to solve the partial differential equation.

A solution to the equation, with respect to y and t is (14)

$$u(y,t) = \frac{4u_0}{\pi} \sum_{1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)} \cos \frac{(2n-1)}{2\ell} \pi y e^{-\{\frac{(2n-1)^2\pi^2 kt}{4\ell^2}\}}$$

For the film, $u(y,0) = u_0 = u_s$.

To find the average velocity in the film,

$$\langle u\rangle = \frac{4u_s}{\pi} \sum_{1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)} e^{-(K)} \frac{\int_0^1 \cos \frac{(2n-1)\pi}{2}(\frac{y}{\ell})d(\frac{y}{\ell})}{\int_0^1 d(\frac{y}{\ell})}$$

if

$$K = \frac{(2n-1)^2\pi^2 kt}{4\ell^2}$$

$$\langle u\rangle = \frac{4u_s}{\pi} \sum_{1}^{\infty} \frac{(-1)^{n-1}}{2n-1} e^{-(K)} \frac{2}{(2n-1)\pi} \sin \frac{(2n-1)}{2} \pi(\frac{y}{\ell}) \Big|_0^1$$

$$= \frac{8u_s}{\pi^2} \sum_{1}^{\infty} \frac{(-1)^{n-1}}{(2n-1)^2} e^{-(K)} \sin \frac{(2n-1)}{2} \pi$$

but

$$\sin\frac{(2n-1)}{2}\pi = (-1)^{n-1} \qquad (n \geq 1)$$

Therefore

$$\langle u\rangle = \frac{8u_s}{\pi^2} \sum_{1}^{\infty} \frac{1}{(2n-1)^2} e^{-\{\frac{(2n-1)^2\pi^2 kt}{4\ell^2}\}}$$

where

$k = \mu/\rho$

t = time interval between slugs

$\simeq$ inverse of slug frequency

ℓ = depth of liquid in the film

$\simeq$ fraction liquid in film to tube diameter

$$= (R_L - \frac{Q_L}{Q_L + Q_G})\, D = \varepsilon D$$

Substitution into the approximated change in momentum equation

$$\Delta P_m = N_s \frac{\rho\varepsilon}{g_c} \{u_s^2 - \langle u\rangle^2\}$$

yields the momentum loss term of the total pressure drop

$$\Delta P_m = N_s \frac{\rho\varepsilon}{g_c} \{u_s^2 - \frac{8^2 u_s^2}{\pi^4} (\sum_1^\infty \frac{1}{(2n-1)^2}\, e^{-\{\frac{(2n-1)^2\pi^2 kt}{4\ell^2}\}})^2\}$$

or

$$= N_s \frac{\rho\varepsilon}{g_c} u_s^2 \{1 - \frac{64}{\pi^4} (\sum_1^\infty \frac{1}{(2n-1)^2}\, e^{-\{\frac{(2n-1)^2\pi^2 kt}{4\ell^2}\}})^2\}$$

To this term is added the friction loss of the slugs

$$\Delta P_f = \frac{2f\rho u_s^2 L}{g_c D} (\frac{Q_L}{Q_L + Q_G})$$

where f is approximated by

$$f = \frac{0.079}{Re^{0.25}}$$

The momentum term is divided by g_c to maintain consistent units.

The overall pressure drop equation for the slug flow model therefore becomes

$$\frac{\Delta P}{L} = \frac{2f\rho u_s^{\,2}}{g_c D}\left(\frac{Q_L}{Q_L + Q_G}\right) + \frac{N_s \rho \varepsilon u_s^{\,2}}{g_c L}\left\{1 - \frac{64}{\pi^4}\left(\sum_{1}^{\infty} \frac{1}{(2n-1)^2}\, e^{-\left\{\frac{(2n-1)^2 \pi^2 kt}{4\ell^2}\right\}}\right)^2\right\}$$

A sample calculation is shown in Appendix C.

CHAPTER IV

EXPERIMENTAL EQUIPMENT

The experimental apparatus, as shown in the schematic flow diagram, Figure 3, and the photographs, Figure 4, consisted of two main parts:

(a) metering system

(b) pipeline.

The design and construction of the equipment was based largely on findings obtained from preliminary investigations carried out on bench scale equipment. This apparatus consisted of a 0.59 inch inside diameter glass tube, about 8 feet long, into which water and air from the laboratory bench were introduced via a mixing tee. Sizing was carried out on the basis of Baker's flow regime plot (4). Aside from using the initial test set-up to study fluid behavior, it was also used for the development of an electronic slug sensor designed to trace out the profile of the individual slugs for possible holdup correlation (40).

The test section, consisting of clear Lucite tubing, of one-half inch diameter, had a total length of 40 feet. The seven sections were joined by flanges, sealed with rubber O-rings. Since no data on entrance length in slug flow were reported in the literature, the first pressure tap was located almost halfway along the tube, 17.9 feet downstream from the mixing tee. This gave an entrance length of 430 pipe diameters. The second pressure tap was located 18.29 feet further downstream, resulting in an exit length of 4.0 feet. The pipeline was mounted on the edge of three laminations

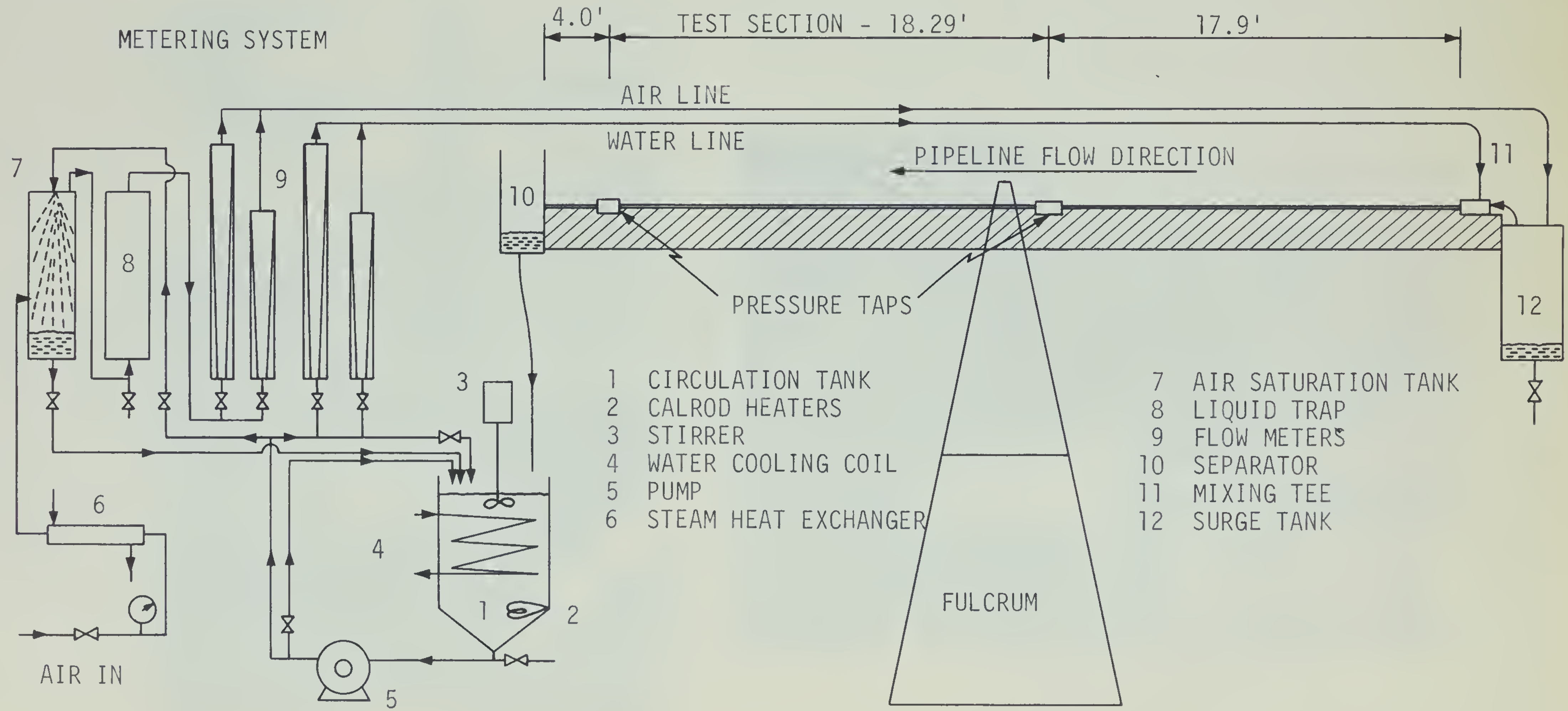
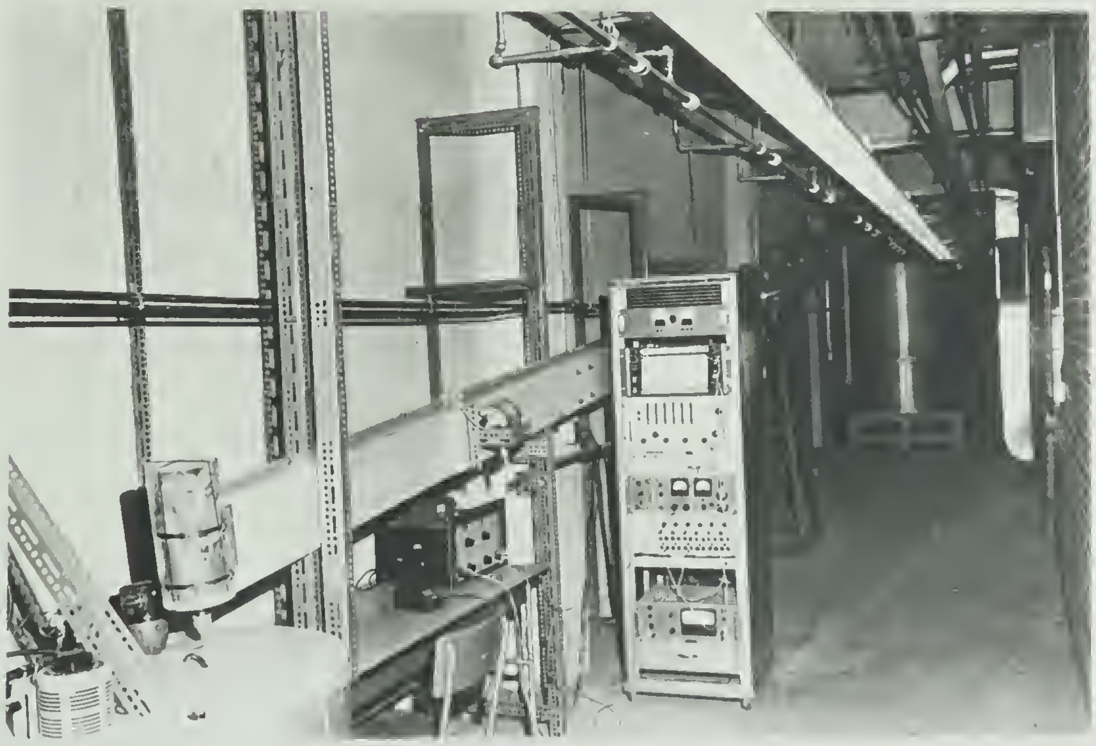


FIG. 3 SCHEMATIC FLOW DIAGRAM OF THE EQUIPMENT

METERING SYSTEM



PIPELINE

FIG. 4 - EXPERIMENTAL EQUIPMENT

of 3/4 inch plywood, and the whole apparatus was suspended on a fulcrum, which was located about halfway along the length for ease in altering the inclination of the tube. For added rigidity, the whole assembly was clamped to rectangular frames fastened to the wall.

In this apparatus the water was circulated in a closed loop, while the air was obtained from the compressed air system and was vented to the atmosphere after passing through the test section. The water circulation was carried out from a 2 foot diameter, 45 gallon, sheet iron tank equipped with a 3/8 inch copper tube cooling coil and two 100 watt Chromalox heaters (Model RI-100). The water was pumped from this tank by a 1/2 horse power Jabsco pump (Model JC-1/2"-37) with 1/2 inch ports. The outlet stream was split up with part going through a bypass, part being used to saturate the preheated incoming airstream, and the remainder going to the flow meters. The flowrate was measured by two Brooks rotameters (Model No. 8-1100 and Model No. 6-1110-GTM). The metered water stream was conducted to the mixing tee, at the far end of the test section, through 3/4 inch copper tubing. The temperature was controlled at 80 ±1 degrees Fahrenheit in the collection tank by means of the cooling coil and heaters. The cooling water was circulated at a constant rate, while heat input to the system was controlled with a Chromalox thermostat (Model AR 1014) and a Powerstat variable autotransformer (Type 116B).

To saturate the entering air at 80 degrees Fahrenheit, it was passed through a steam heated Heat-X (Model 120) heat exchanger where its temperature was raised to 178 degrees Fahrenheit for adiabatic saturation at 80 degrees (9). Quenching to 80 degrees was carried out in a special tank into which part of the circulating water stream was

sprayed through a nozzle. The saturated air then passed through a liquid trap and to two Brooks Rotameters(Model No. 8-1100 and Model No. 6-1110-GTM). The metered air was then passed to the mixing tee through 1/2 inch copper tubing. The flow to the various flow meters and tanks, downstream of the pump, was controlled with Crane 1/2 inch needle valves (Cat. 88). The two fluids were introduced to the test section through a mixing tee, a chamber about 1-1/4 inches long and 3/4 inches in diameter, with the air and water entering at right angles to each other. Two 125 mesh stainless steel screens were placed downstream from the mixing tee for improved mixing and to cause a pressure drop to reduce the effect of pressure surges in the test section. Pressure surges and backflow of the water were also controlled by the surge tank located ahead of the mixing tee.

Pressure drop and slug presence were determined by two Statham (Model No. PM80TC ±15-350, rated for ±15 psid at 5000 psig) differential pressure transducers, connected to the test section by means of short, water filled, 3/16 inch Polyflo tubing. Excitation of these transducers and amplification of the signal from them was carried out by Hewlett-Packard (Sanborn Model 8805A) carrier pre-amplifiers, operated according to the instruction manual (8805A). The two signals were recorded simultaneously on a Hewlett-Packard (Moseley Autograf Model 7100 B) two channel stripchart recorder.

Slug frequency and velocity were measured by various means. These data were primarily taken from the strip charts, however, visual and electronic checking was done over most of the range. Low slug frequencies and velocities were measured visually with the aid of a stopwatch. At higher frequencies the slugs were sensed with a photo-

conductivity cell (Clairex CL5M7) and counted on a Beckman (Model 7350R) Universal EPUT and Timer.

After passing through the pipeline, the air and water were allowed to separate with the air venting to the atmosphere, and the water returning to the circulation tank. To reduce the effect of the pressure surges, caused by the slugging, the separator tank was packed with wire mesh screen to dissipate the energy of the slugs.

CHAPTER V

EXPERIMENTAL PROCEDURE

After a steady temperature of 80 degrees Fahrenheit was attained in the circulation tank, the air was turned on, as well as the steam to the air heater. The heater's temperature was adjusted to 180 degrees by manipulation of the inlet and outlet throttling valve. Then the pump was switched on, allowing the water to regain steady temperature by circulation.

In the meantime the electronic apparatus was allowed to warm up, calibration of the preamplifiers was checked, and the strip chart recorder was zeroed. When steady temperatures were regained throughout the system, the valves to the air rotameters were opened first. Then the water rotameters were opened. This order prevented flooding of the air system with water. Finally, the water line to the spray tank was opened and the needle valve at the bottom of this tank was coarsely adjusted to maintain a liquid trap in this vessel. Testing was ready to begin.

Data were taken by holding the air rate at a constant level, and advancing the water rate through the region of slug flow. This procedure had several advantages over holding the water rate steadily and advancing the air rate. It was found, as may be confirmed from Baker's chart, Figure 2, or from the appearance of flow patterns according to Alves (1) that if the water rate were held constant, and the air rate varied, several flow patterns had to be passed through before the slug flow pattern was reached. With a steady air rate and varying water rate, however, only stratified or wavy flow had to be

passed. An additional reason for using the preferred procedure was that the needle valves to the rotameters were not precision made and were therefore affected by vibration and pressure on the stems, causing the valves to close gradually and therefore required almost constant valve manipulation to maintain a steady water rate. This effect was not noted with the valves to the air rotameters, although the same type of valve was employed.

The pressure levels and fluctuations from each of the two pressure transducers were recorded on the two-channel strip chart recorder. Checks of slug velocity and frequency were made by timing the passage of a slug between the pressure taps and by counting the number of slugs in a certain time interval, respectively. Slug frequency was further determined by measuring the change in resistance in a photoconductivity cell, mounted on the test tubing, when a slug of liquid passed in front of it. To amplify the change in resistance the water in the system was dyed with Flourescein dye. The signal was filtered for line frequency, and the slugs, represented by oscillations, were counted on the Beckman EPUT Meter.

CHAPTER VI

INTERPRETATION OF RESULTS

6.1 Flow Patterns

The transition between flow patterns at a constant liquid rate with increasing gas flow rate was observed to occur as described by Alves (1) and as is shown on Baker's chart, Figure 2. With an increasing air rate and a low water rate, the order of patterns observed was stratified, wavy and annular. At a somewhat higher water rate, stratified, plug, slug and annular flow occurred. At the high liquid rates the order of bubble, slug, and annular flow was followed. Mist flow was not observed due to pressure limitations of the compressed air system.

No clear distinction appears to exist between plug and slug flow, as evidenced by the dotted transition line on Baker's area plot, Figure 2. At low fluid rates the water tended to bridge the tube cross section, forming a fairly regular series of long, quietly moving water plugs. With an increased air rate the rate of slug formation became somewhat more erratic and the slug length became quite variable. Froth began to form at the front of the slugs. At the highest air rates the whole slug consisted of a frothy mass but was still recognizable as a slug by its stability and high velocity. A further increase in air rate caused the collapse of slugs with the subsequent beginnings of semi-annular flow.

The range of the slug flow pattern that was studied in this report has been superimposed on the flow pattern map of Baker (4). The regions covered by each of the three tube inclinations are shown

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

in Figures 5 to 7. The region in which irregular slug flow occurred, i.e., slugs were formed infrequently and often broke up soon after formation, was marked as "Intermittent Slugging" on these plots. This area was also found to be located largely outside of the slug flow region denoted by Baker (4). It is suspected that the entrance conditions are responsible for this phenomenon. Hoogendoorn (24) noted that by using different entrance conditions it was possible to obtain either slug or wave flow the other conditions remaining the same. Furthermore, Brigham (7) stated that he was unable to produce stratified flow due to the return bends which caused the liquid to bridge the tube cross section. A further observation made by this author was that in essentially wavy flow the slugs leaving the outlet of the pipeline appeared to generate new slugs at the mixing tee. It was also possible to create slug flow by momentarily plugging the exit to cause a pressure buildup. The sudden release in pressure with removal of the obstruction caused the liquid to bridge and slugs were generated. Scott (41) summarized that there is a possibility of two distinctly different flow patterns occurring, depending on such factors as the inlet arrangement. Although one of these patterns may be unstable, its transition to the more stable configuration may not be observable within the limited scale of an experimental apparatus. McDonald (32) also verified this point in describing the effect of water and air being fed into a 75 foot long experimental pipeline of 1.25 inches diameter. The flow pattern was observed to change from stratified to wave, and then slug flow. While this is thought to be primarily the effect of the increasing gas volumetric fraction due to the gas expansion along the pipeline it shows that the flow pattern can be highly unstable.

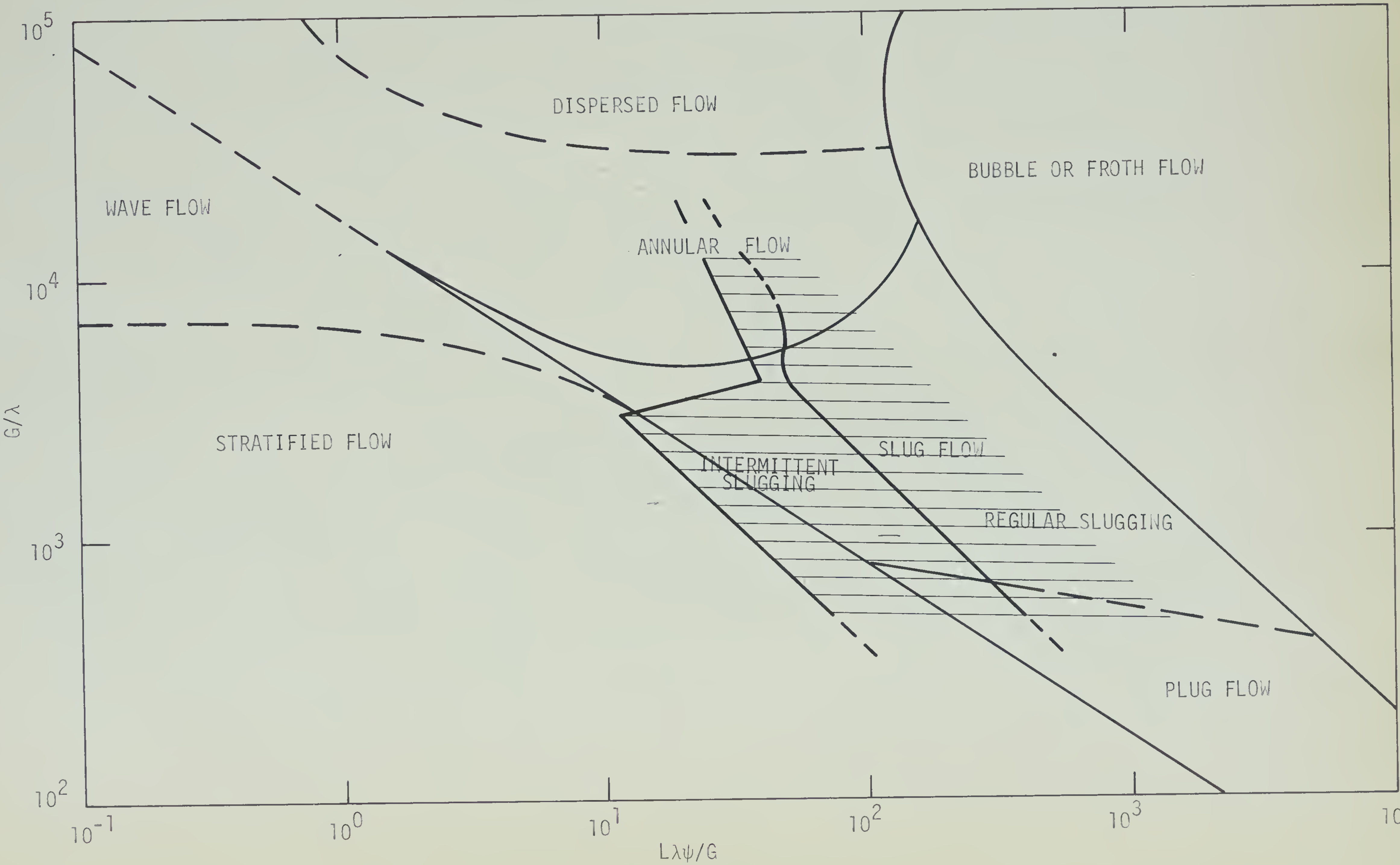


FIG. 5 - COMPARISON OF BAKER'S FLOW PATTERN CORRELATION WITH THE DATA IN HORIZONTAL FLOW

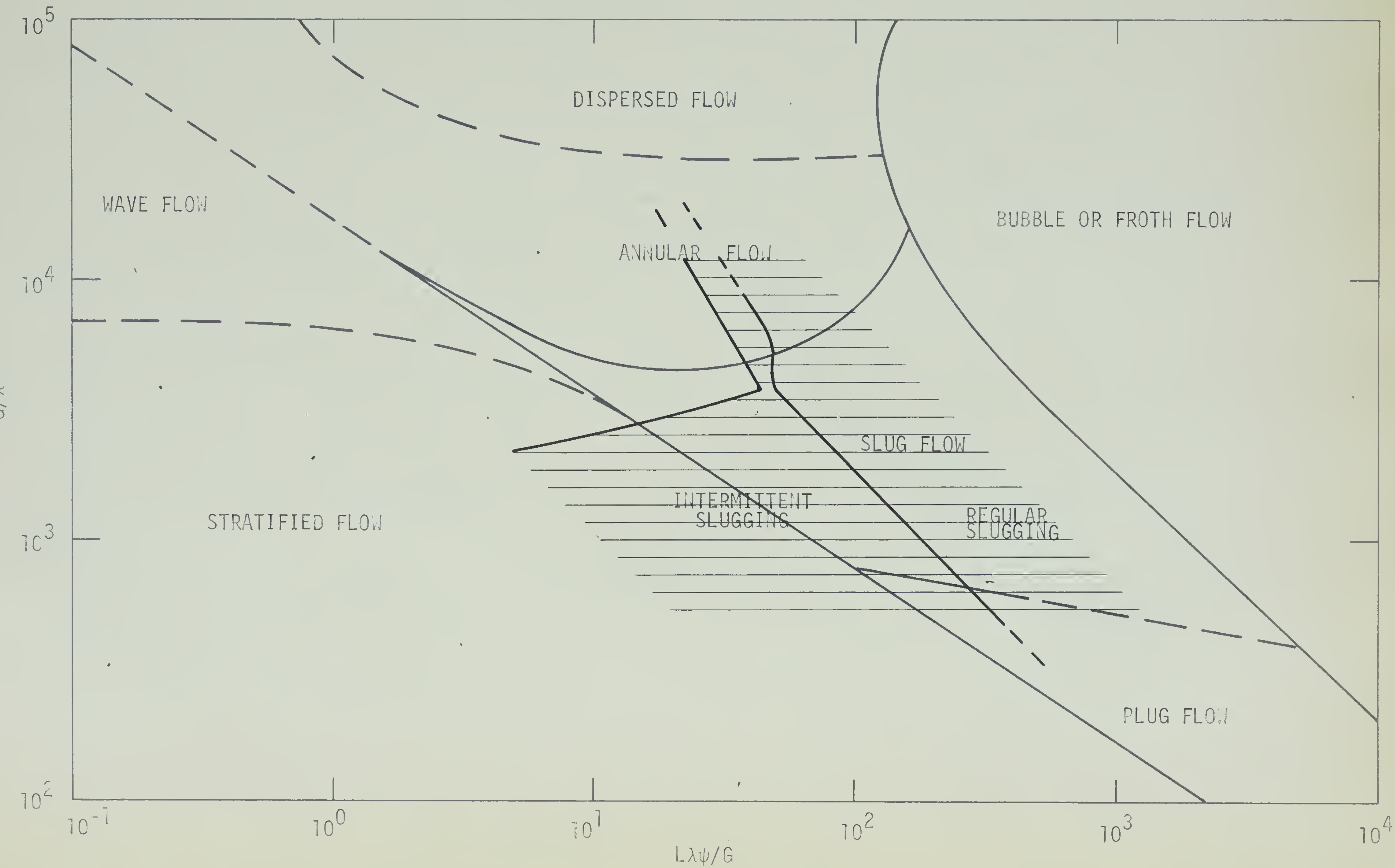


FIG 6 - COMPARISON OF BAKER'S FLOW PATTERN CORRELATION WITH THE DATA IN POSITIVELY SLOPED FLOW (+7°)

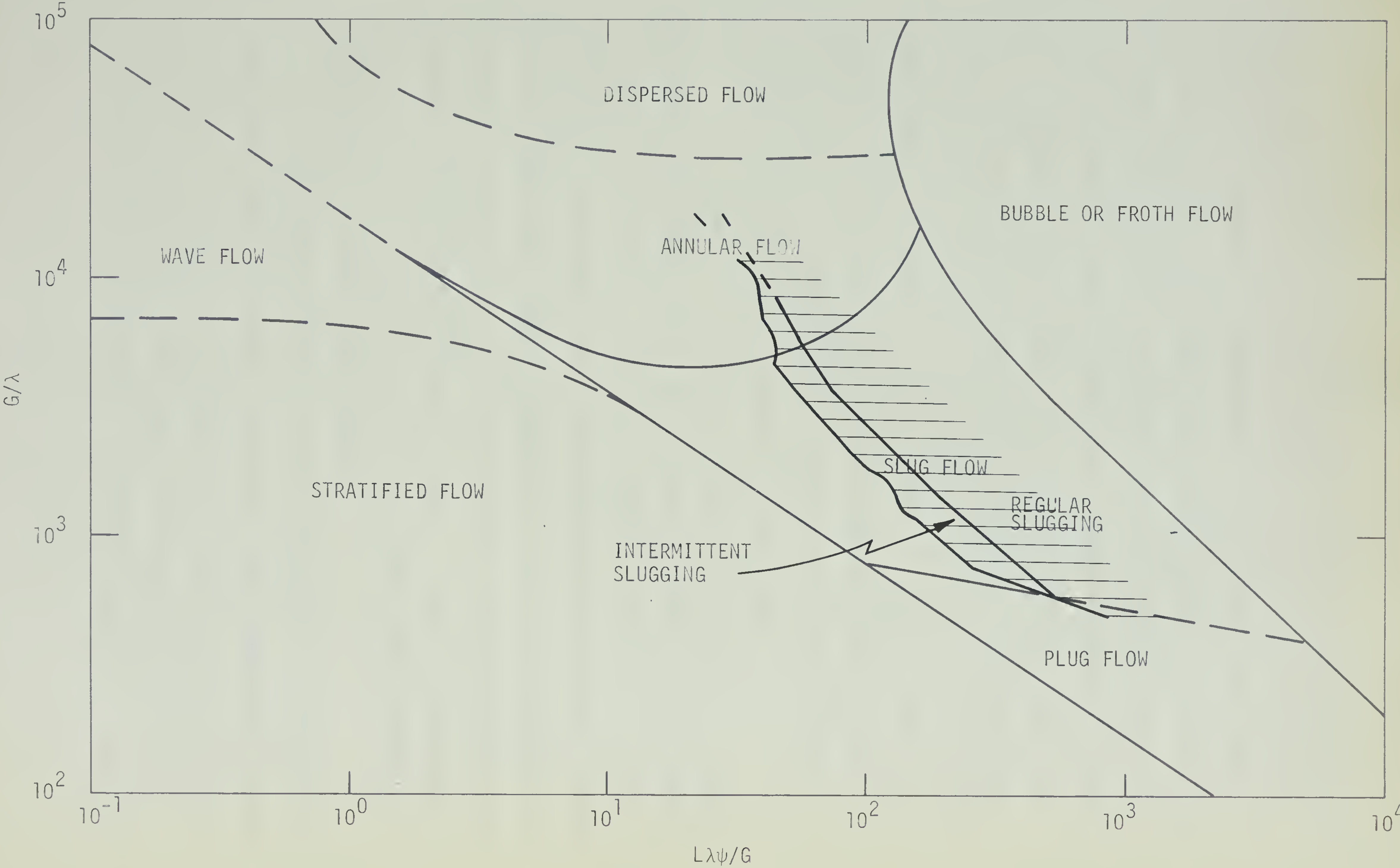


FIG. 7 - COMPARISON OF BAKER'S FLOW PATTERN CORRELATION WITH THE DATA IN NEGATIVELY SLOPED FLOW (-7°)

In inclined flow Kosterin (29) noted that the entrance conditions exerted only a very small influence on the areas of stable flow structures. Figure 6, for positively inclined flow, shows, however, that the slug flow area is even further enlarged than was the case for horizontal flow, Figure 5. The reason for these contradictory observations is probably due to the fact that Kosterin's tube was inclined at 70 degrees to the horizontal, virtually vertical flow, while the pipe of this work was inclined to only 7 degrees. The intermittent slugging noted in horizontal flow, which was primarily caused by slug breakup, was still evident. No stratified flow patterns could be observed in this tube inclination, as confirmed by Brigham (7) and Kosterin (29). It is reasonable to conclude that gravity effects preclude stratified flow in positively inclined tubes.

In downward sloping flow the region of intermittent slugging was found to be almost non-existent, primarily caused by slug breakup occurring before the test section was reached. The region of stratified and wavy flow became greatly enlarged and slug flow did not occur until the higher mass rates were employed. Under the circumstances of negatively inclined tubes, it can be concluded that the separated flow patterns would be favored at low fluid rates. It must be emphasized, however, that this observation may not be true for tubes inclined at angles greater than minus seven degrees.

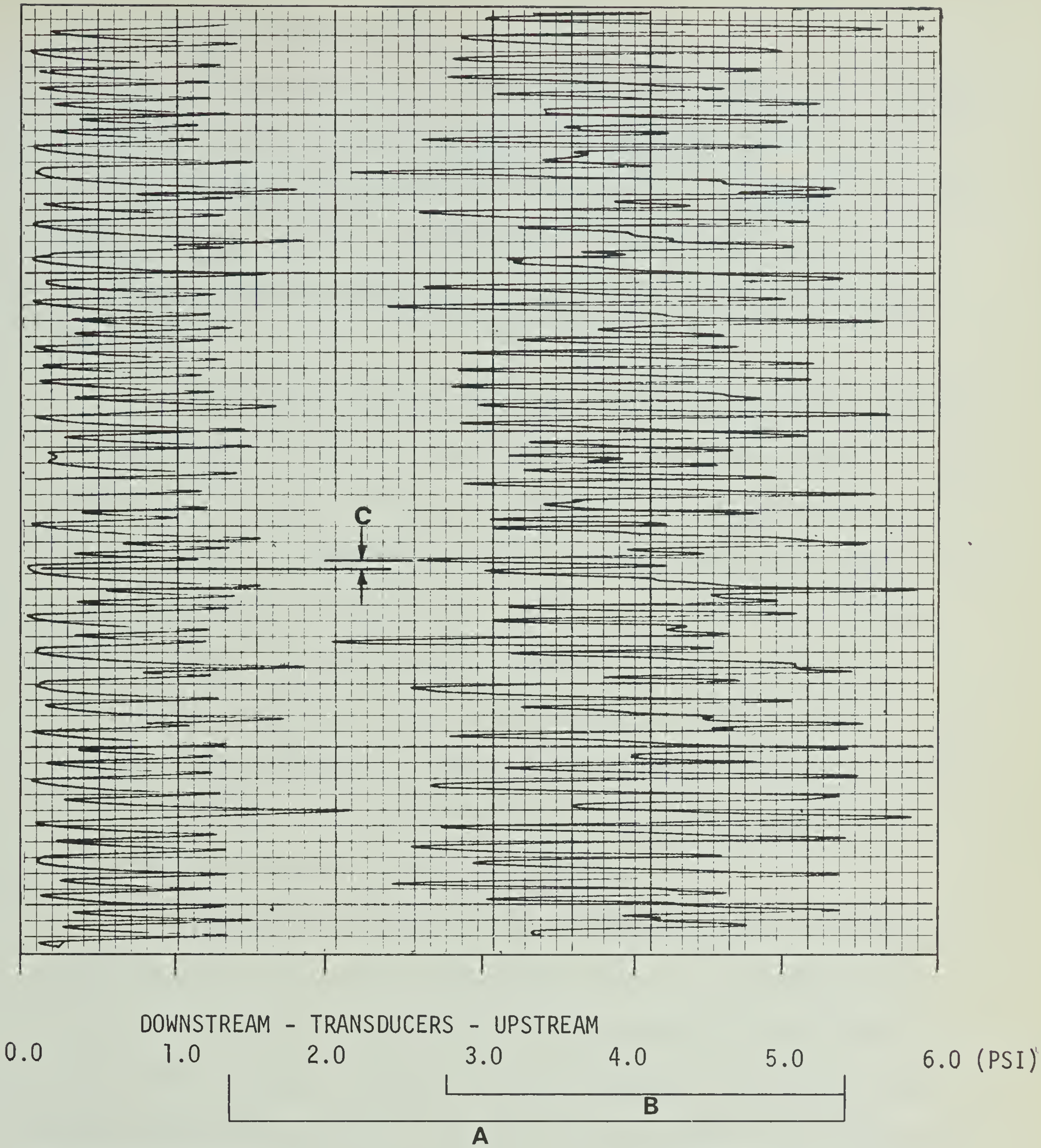
A comparison of the transition lines between intermittent and regular slug flow for all three inclinations shows them to be virtually coincident. To forestall the argument that these lines may have been subject to operator bias, measurement of the range of slug flow was carried out by two separate means, and identical areas, for both types

of slug flow, were obtained. In the first investigation the water rate was held constant while the air rate was varied for each setting of water rate. In the second run the air was held constant and the water rate was advanced, again through the complete range of both fluids. Each test run was carried out independently of the other and the plots of the two sets of data were found to coincide. It must therefore be concluded that in regularly slugging flow, the angle of inclination of plus and minus seven degrees does not alter appreciably the area for stable slug flow, if shown on Baker's (4) plot. This observation has not been reported in the literature for slug flow. However, Brigham (7) found that "the angle of slope definitely does not affect the overall pressure drop when the fluids are flowing in wave, cresting, and semi-annular flow". This would, of course, neglect the effect of static head. Since pressure drop is directly related to flow patterns, regularly slugging flow must be included in this grouping of Brigham's.

The actual data for this work were taken only in the regular slug flow region as the area of intermittent slugging may have been the result of the equipment characteristics, as discussed above.

6.2 Slug Flow Characteristics

A typical example of a strip chart recording is shown in Figure 8. The pertinent reference lines have been added to show how the measurement for the data points was carried out. Pressure drop, pressure fluctuations, slug frequency and slug velocity, which was calculated from the average time taken for a slug to travel the length of the test section, were read off the charts. Checks on velocity and frequency were made visually and electronically.



TUBE INCLINATION +7°
CHART SPEED 2.0 DIV/SEC
CALIBRATION 0.103 PSI/DIV

FLOW RATES	
AIR(CFM)	WATER (USGPM)
1.475	1.16

DATA FROM CHART

A - PRESSURE DROP 4.0 PSI
B - PRESSURE FLUCTUATION 2.5 PSI
C - SLUG TIME 0.75 SEC
SLUG FREQUENCY 110/MIN

NOTE: UPSTREAM PEN IS OFFSET BY 1.0 DIV (0.5 SEC)

FIG. 8 - STRIPCHART RECORDING

6.2.1 Overall Pressure Drop

The measurements of total pressure drop are shown in Figures 9 to 11. In comparing these curves with respect to tube inclination it must be noted that the inclined tubes were not corrected for static head, which is a function of holdup (21). As a result, only the relative magnitudes and slopes of the individual curves are discussed below.

The figures show the strong influence of gas rate on overall pressure drop. At the lowest air rates the curves are found to be quite flat and an increasing liquid rate causes only a gradual increase in pressure drop. An increased air rate, however, causes the slope of the curves to grow much more rapidly. There is an upper limit, however. As the maximum rates were attained, the slug flow pattern changed to semi-annular and annular flow since slug breakup began to occur. The effect of this phenomenon is shown in the gradual decrease in slope of the upper curves. It is unfortunate that equipment limitations prevented further data points from being taken.

A comparison of the results of the three tube inclinations shows the curves to be very similar with respect to location on the chart and to slopes. The offsets are due primarily to the effect of static head, however, they may also be influenced by the lag in movement of the recorder pens as well as by operator bias. The difficulty in taking measurements off the charts lies with the often extremely violent pressure fluctuations characteristic of slug flow. The recorder pen lag would have reduced this effect somewhat, and therefore could have caused the readings to be in error by several percent. This problem could be overcome in part through the use of a recorder with

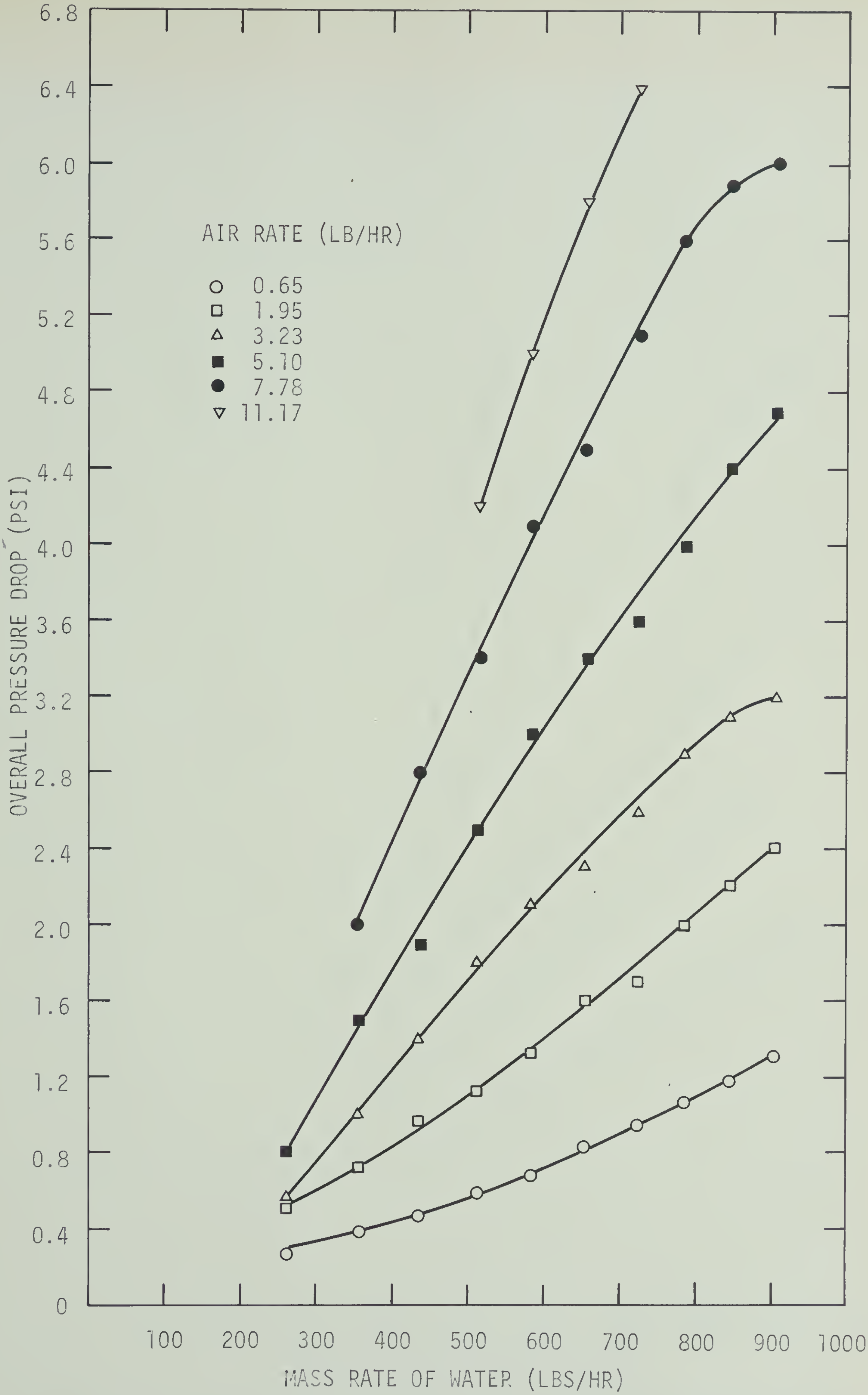


FIG. 9 - OVERALL PRESSURE DROP FOR HORIZONTAL FLOW

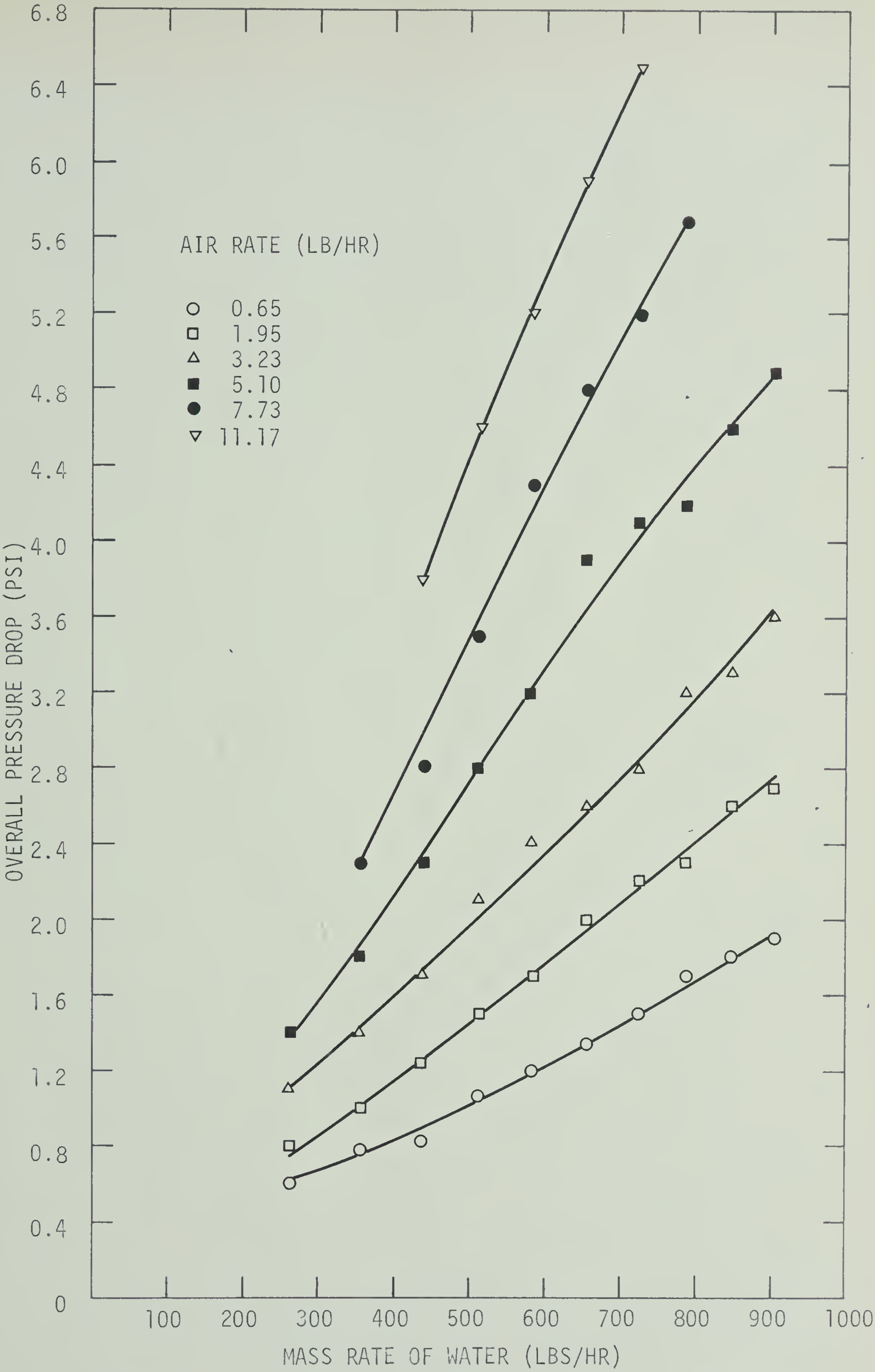


FIG. 10 - OVERALL PRESSURE DROP FOR POSITIVELY SLOPED FLOW (+7°)

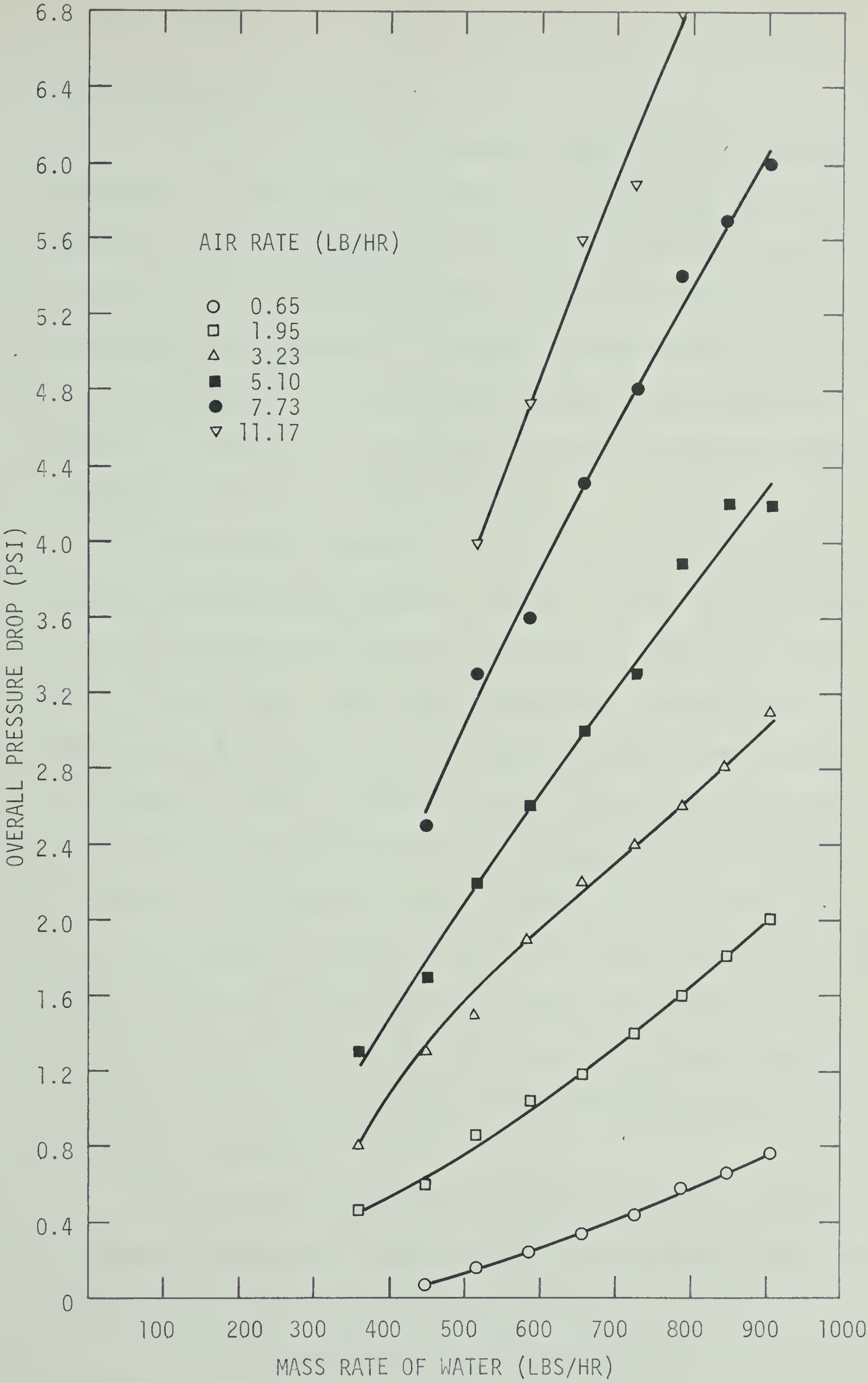


FIG. 11 - OVERALL PRESSURE DROP FOR NEGATIVELY SLOPED FLOW (-7°)

a faster response.

6.2.2 Pressure Fluctuations

Figures 12 to 14 show the magnitude of slug flow pressure fluctuations. Although this phenomenon has been noted by almost all investigators with reference to slug flow, Kordyban (28) was the first to publish data on such measurements. The use of differential pressure transducers makes it possible to record the fluctuations, while this was not possible with the previously employed U-tube manometers. Brigham (7) noted that it was necessary to design special buffers in the manometer lines to reduce this effect.

The figures show that at low gas rates with increasing water rate the pulsations are virtually constant. Again, as was the case for overall pressure drop, the gas rate shows a greater influence than does the liquid rate. The slopes of the curves increase sharply with increased gas rates, but again the gradual change to semi-annular flow is noticeable. With a constant air rate and increasing water rate a maximum is obtained, after which the fluctuations are reduced and tend to level off. The pressure surges of the maximum air rates are actually found to be lower than those of the next to highest air rates. The transition from very slow slug flow, commonly referred to as plug flow, through normal slug flow and to semi-annular flow can be seen on these figures by the changes from very low fluctuations through the maximum and the final decrease.

No plausible explanation could be found for the fact that the maxima in horizontal flow were reached at much higher water rates than for either of the two tube inclinations.

The gross magnitude of these pulsations has never been re-

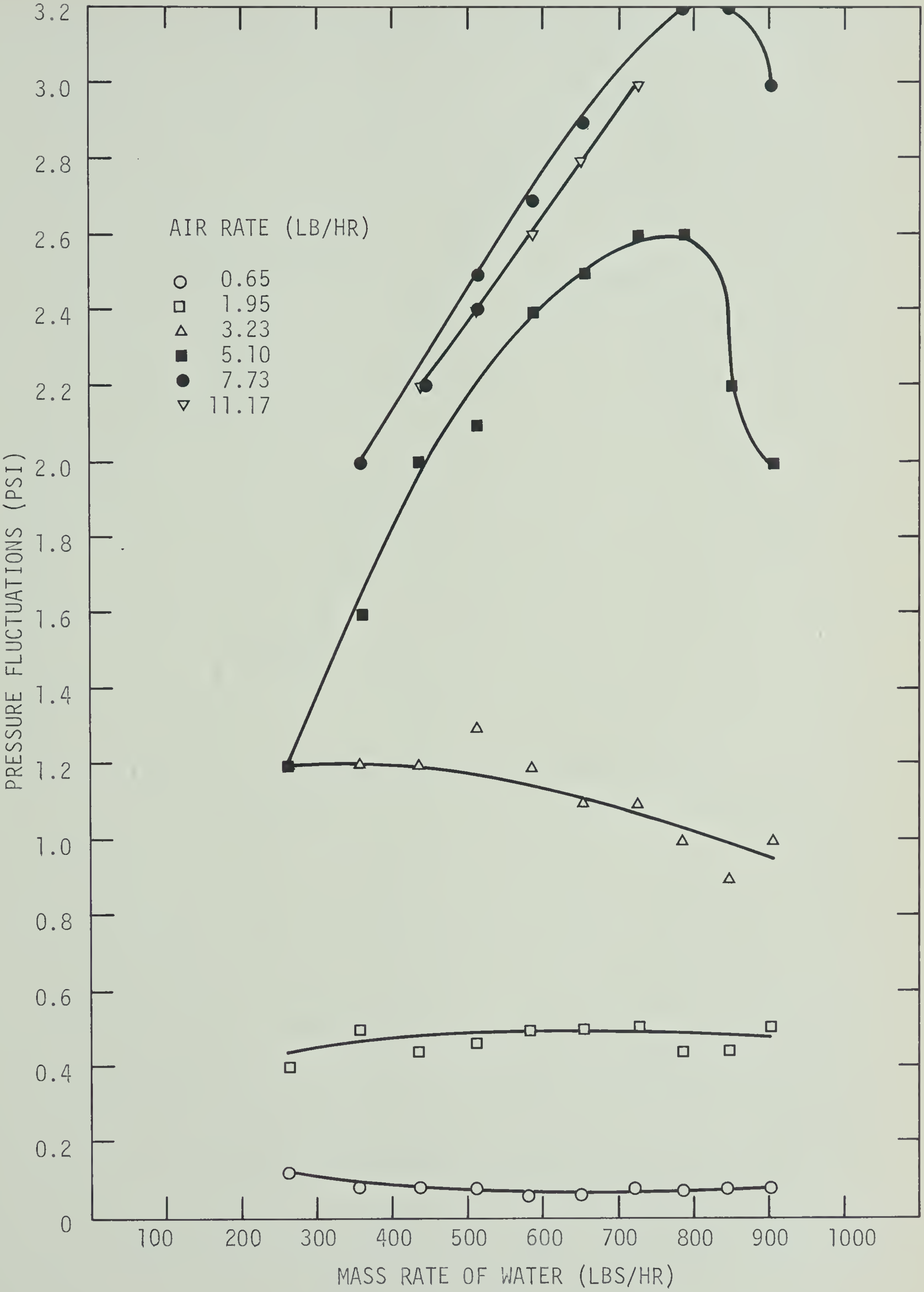


FIG. 12 - PRESSURE FLUCTUATIONS FOR HORIZONTAL FLOW

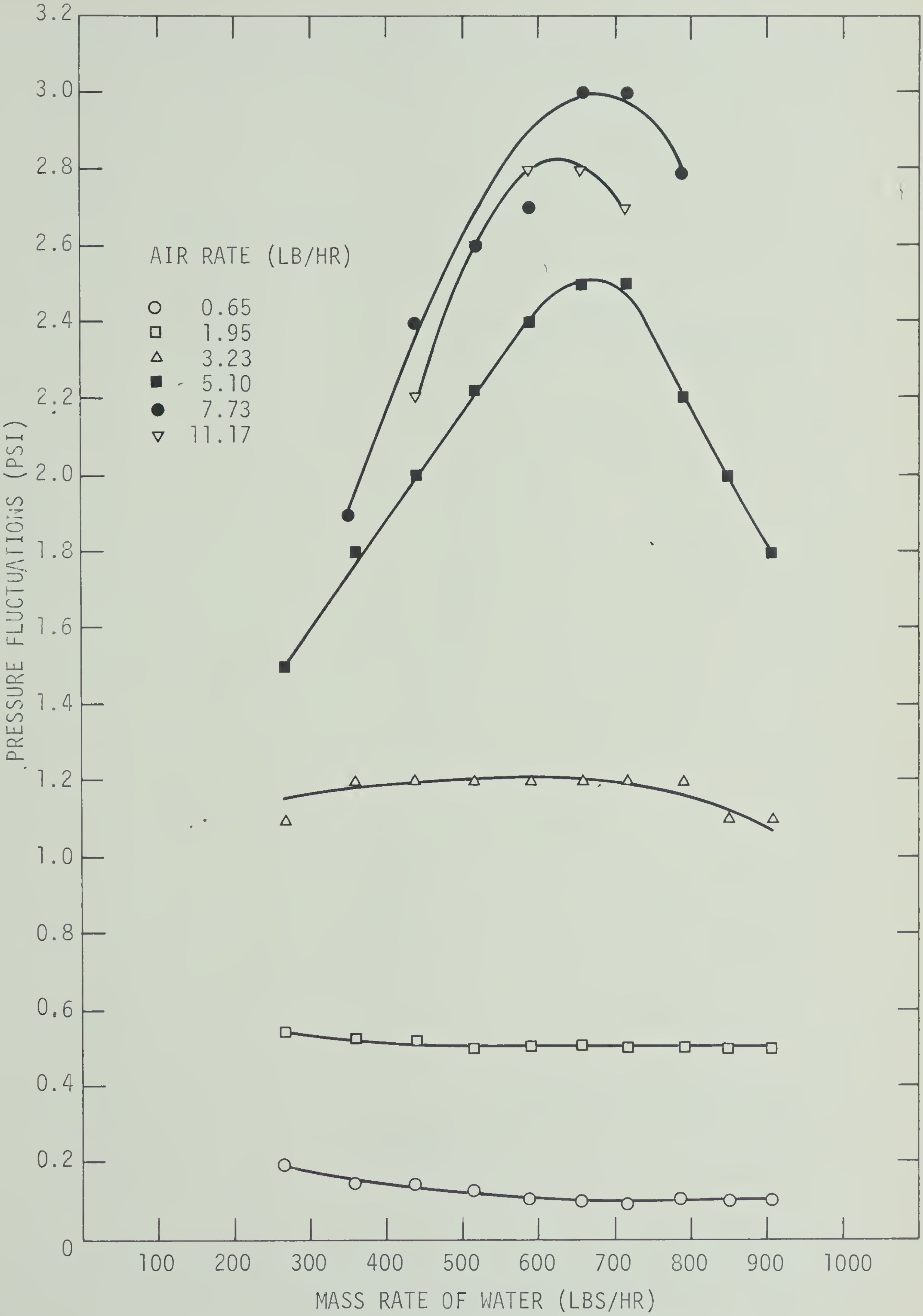


FIG. 13 - PRESSURE FLUCTUATIONS FOR POSITIVELY SLOPED FLOW (+7°)

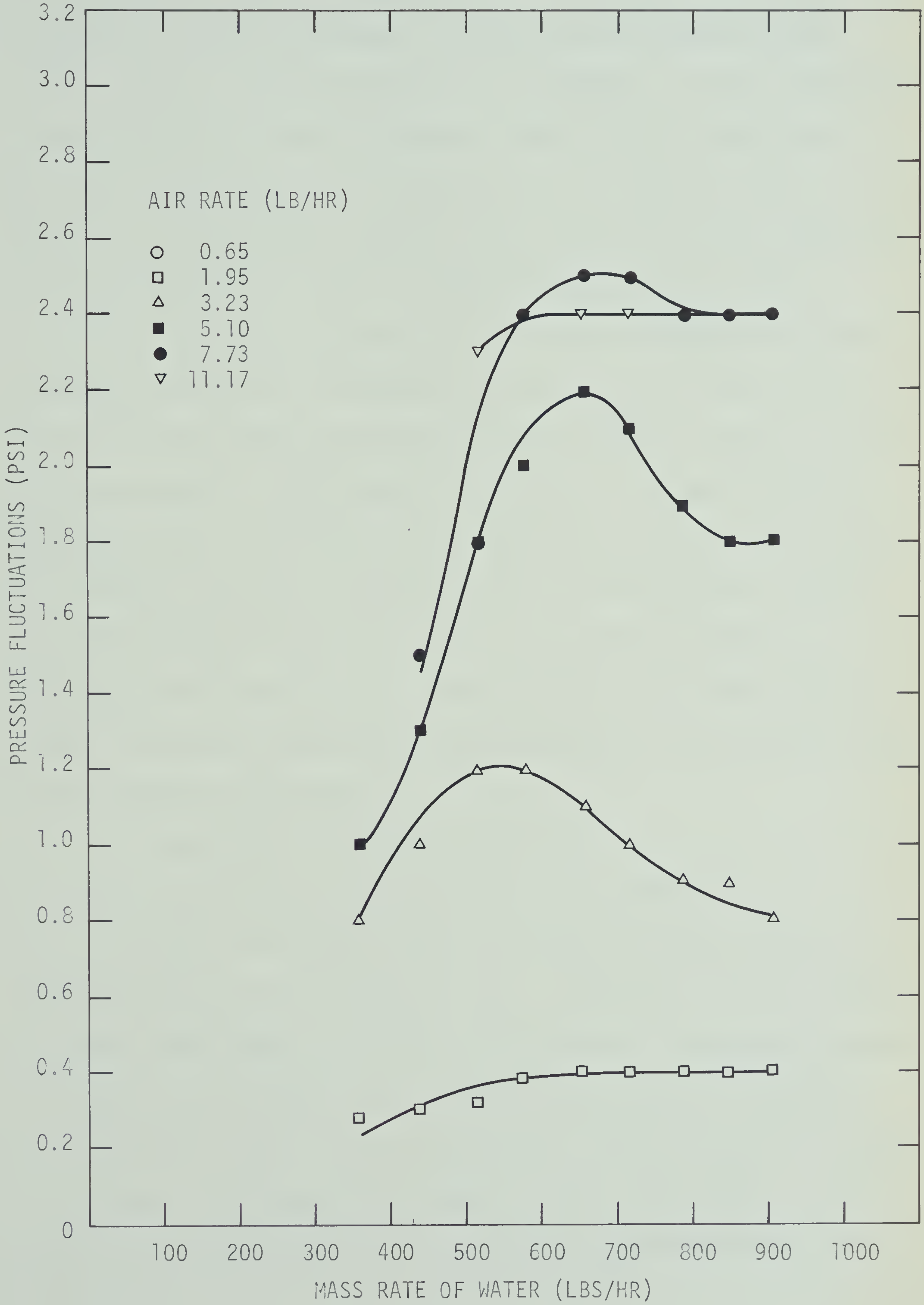


FIG. 14 - PRESSURE FLUCTUATIONS FOR NEGATIVELY SLOPED FLOW (-7°)

ported in the literature, although it is normal practice for multi-phase gathering systems to employ a buffer at the terminals to prevent damage to the receiving vessels. These excessive pressure surges could, however, also be found within the pipeline. In this investigation it was necessary to pack the separator tank with wire mesh screens to dissipate the energy of the slugs, which caused severe vibrations to the equipment at high rates. Figures 12 to 14 show that these surges were in the order of 25 to 30 percent of the overall pressure drop.

A tentative explanation of the relative magnitude of these pressure fluctuation is presented in Figure 15. Four views of the motion of slugs within a pipe are shown with respect to time. In the initial situation three slugs are in the pipe. The second sequence shows the slugs to have advanced to the right by one-half slug length and the third and fourth view shows them to have advanced a further two-half lengths. A new slug appears at the left in sequence 4. In the second view one-half of the right-most slug has already left the tube. To consider the pressure profile along the tube, two main simplifying assumptions were made:

1. Acceleration effects are negligible
2. Pressure drop due to the gas is negligible

Each of the slugs in the tube causes, in this case only, a constant pressure drop across each slug. In the lower part of Figure 15, then, path number 1 is followed from point A. Each slug represents a certain pressure drop, while the gas-filled voids are represented as a constant pressure line from the front of the left slug to the tail of the next. The gauge pressure goes to zero at the outlet. In the second view of the pipe half of the right slug has already left the

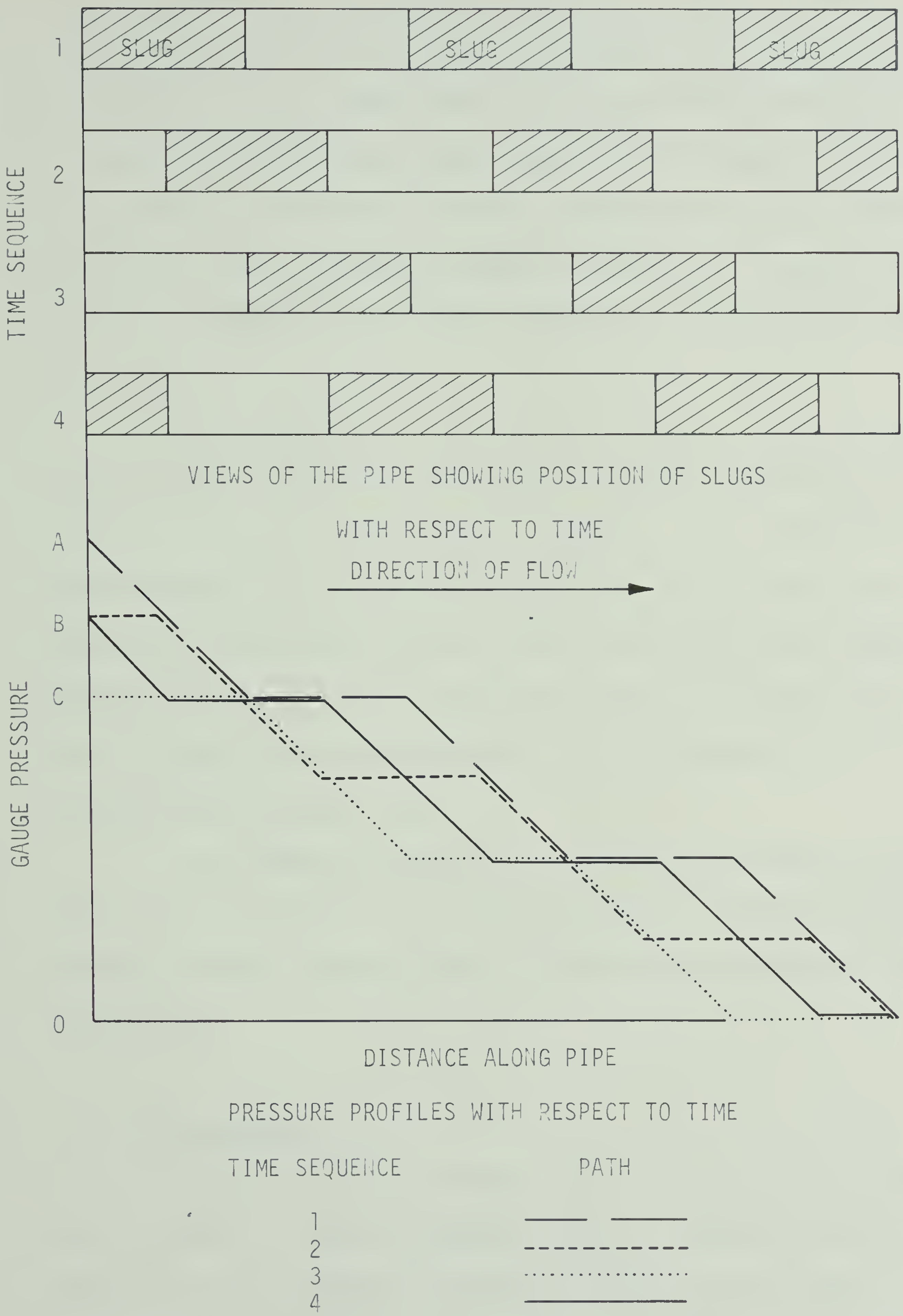


FIG. 15 MODEL OF PRESSURE FLUCTUATIONS IN SLUG FLOW

tube. The pressure B at the left is therefore lower by an amount equal to the pressure drop of the half slug that has left and path 2 is followed. The sequence is continued through view number 3 in which the pressure at the left is at C, lower than A by one whole slug's pressure drop. In view 4 the pressure has again risen to point B since a new half slug has appeared at the left. This whole sequence from 1 to 3 repeats itself then as long as new slugs appear at the left. Therefore the pressure fluctuations cycle from a maximum of A through the minimum of C.

The magnitude of these pressure surges, if the above assumptions are still valid, would be approximately constant, whether the outlet pressure is at zero or at a positive pressure. The differential pressure would therefore be constant for either case, while, the pressure surges, relative to the outlet pressure, could be negligible with respect to a high exit pressure. For a low outlet pressure, as in this work, they could, however, be quite substantial, i.e., in the order of 25 to 30 percent of the pressure drop.

No attempt has been made to correlate the data with this model. It also remains to be shown what effect different fluid properties and tube sizes would have on the relative magnitude of these fluctuations.

6.2.3 Slug Velocity

The slug velocity curves, shown in Figures 16 to 18, indicate that the angle of tube inclination, within ±7 degrees from the horizontal, has little effect on either the shape of the curves or on the magnitude of the velocities. As was the case for pressure drop at low

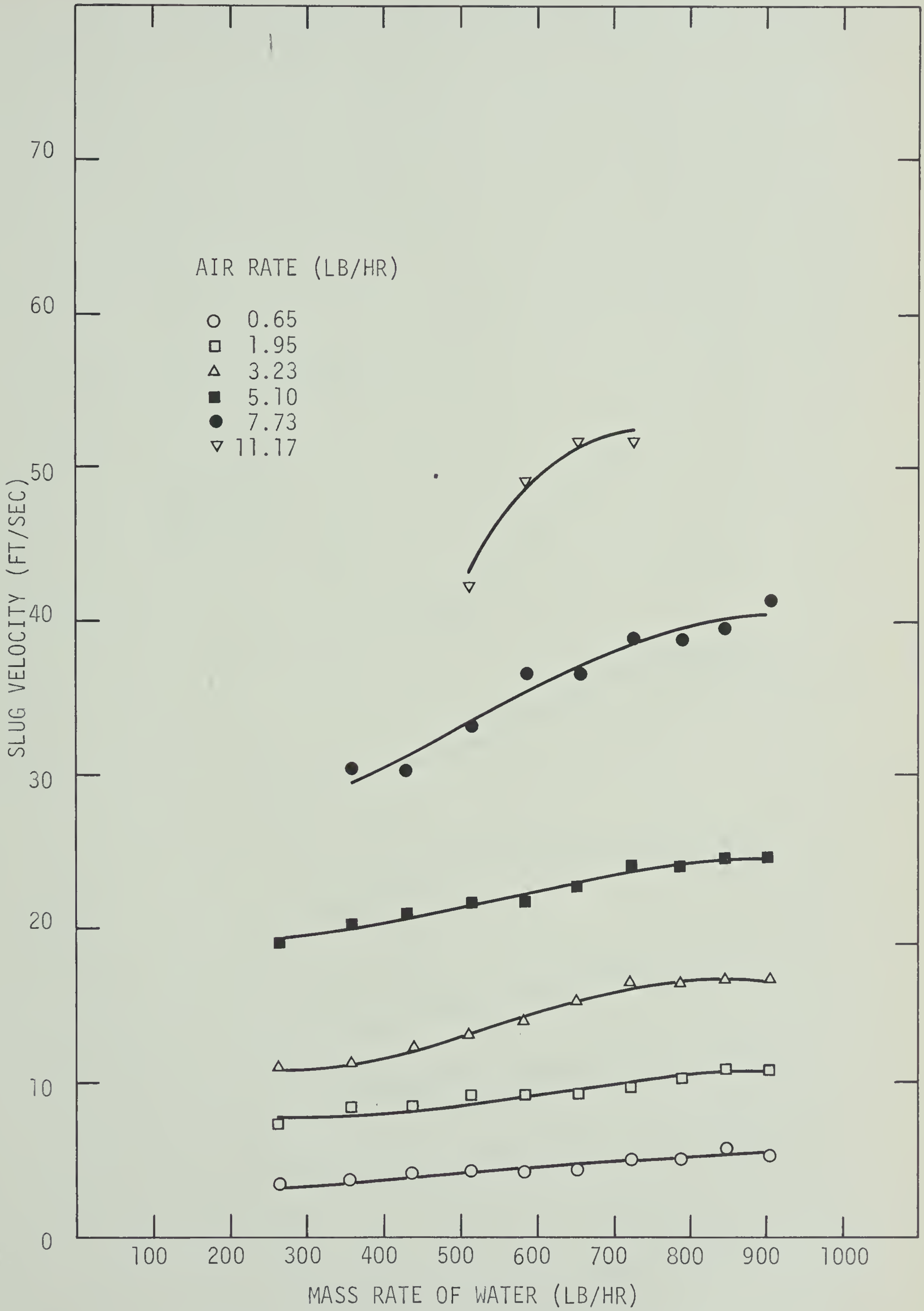


FIG. 16 - SLUG VELOCITY FOR HORIZONTAL FLOW

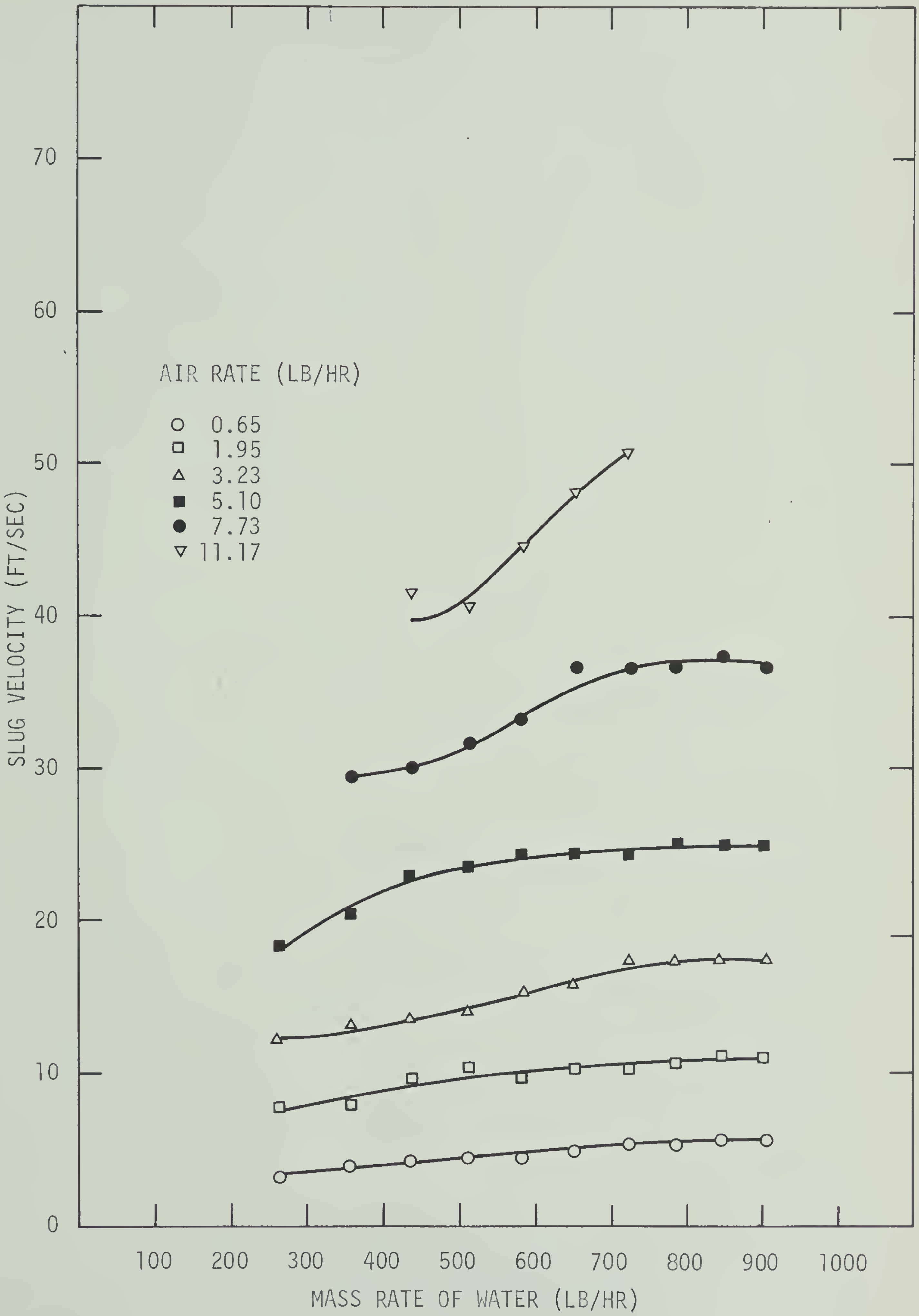


FIG. 17 - SLUG VELOCITY FOR POSITIVELY SLOPED FLOW (+7°)

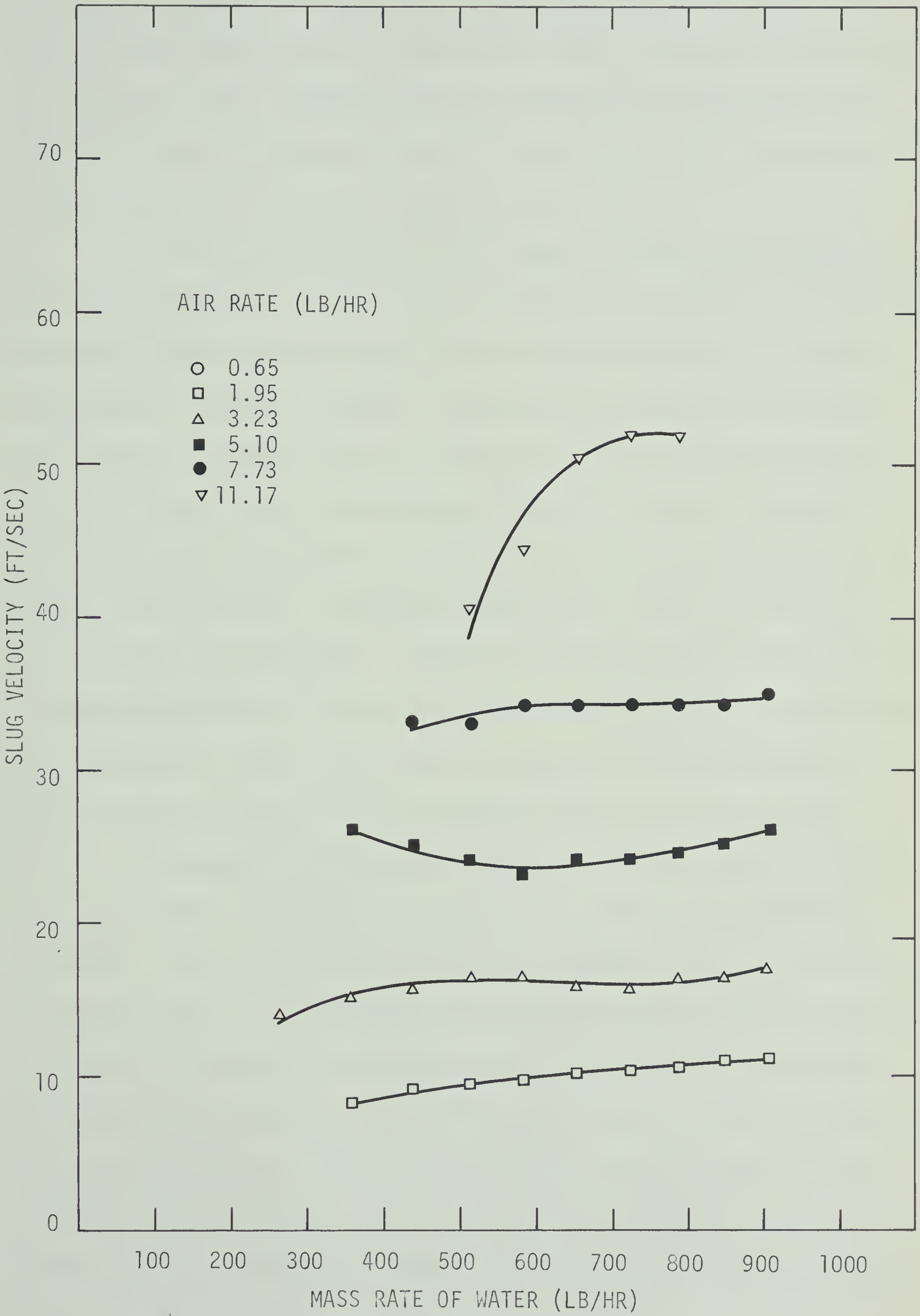


FIG. 18 - SLUG VELOCITY FOR NEGATIVELY SLOPED FLOW (-7°)

air rates, an increasing water rate has a lesser effect on the change in magnitude then does the air rate. It is found that the slopes of the curves rise sharply with an increasing air rate. Again, as in the case for pressure drop, a maximum slope is reached with the highest fluid rates after which a decrease occurs. This effect is also attributed to the transition from slug to semi-annular flow.

The velocities shown on the graphs represent an average of the velocities of individual slugs. Velocity was found to be quite variable and appeared to be strongly influenced by slug size, as would be expected by a material balance. Although both methods for measuring the average time required for a slug to travel the length of the test section-direct timing, and reading off the strip charts - agreed fairly well, a distinct scatter of the data points can be seen on the velocity curves. Nevertheless, the general shape of the curves is similar for all three tube inclinations. Comparison with Kordyban's (28) results is difficult because his work was carried out at lower fluid velocities, but the trend is again very similar. The air rate also appeared to have a greater influence on slug velocity than did the water rate.

Kordyban (27, 28) was not able to determine acceleration of the slugs since his work was carried out at essentially atmospheric pressure. The pressure drop in this work, however, rose to a maximum of over six psig, so that acceleration effects would certainly not be negligible. By taking the effect of the liquid film in reducing the available cross sectional area for flow into consideration, the measured slug velocity was still found to be up to 50 percent higher than the calculated fluid velocity. But slug velocity is directly proportional to the pressure differential across the slug so that the gas expansion,

caused by an overall pressure drop of over 6 psig, could accelerate a slug by over 25 percent. Slug acceleration was, however, not measured in this work.

A further effect on increased velocity could be the effect of the froth at higher rates. The froth was observed to increase the thickness of the film as well as to increase the size of the slugs so that, at higher rates, the film may no longer be completely picked up by the slugs. This would cause a restriction in the tube diameter and result in higher slug velocities. The possibility that the hold-up predictions by Lockhart-Martinelli's (30) correlation are too low could also be a cause for the apparent discrepancy between calculated and measured velocities. It would appear, however, that slug acceleration would cause the major contribution. Kordyban (27) noted that acceleration due to increasing amounts of vapor in his steam-water system could be calculated to be over 50 percent of the total pressure drop, even though his work was carried out at essentially atmospheric pressure.

6.2.4 Slug Frequency

The effect of fluid rates and tube inclination on slug frequency is shown in Figures 19 to 21. At low gas and liquid rates the frequency is low. It was observed that the slugs were very long, about 12 to 18 inches, and moved at a low velocity. As the air rate was increased, the frequency did not change appreciably. At very low air rates and an increasing water rate, however, the frequency was greatly increased, while a steadily increasing air rate caused the frequency to drop and then level off for each water rate.

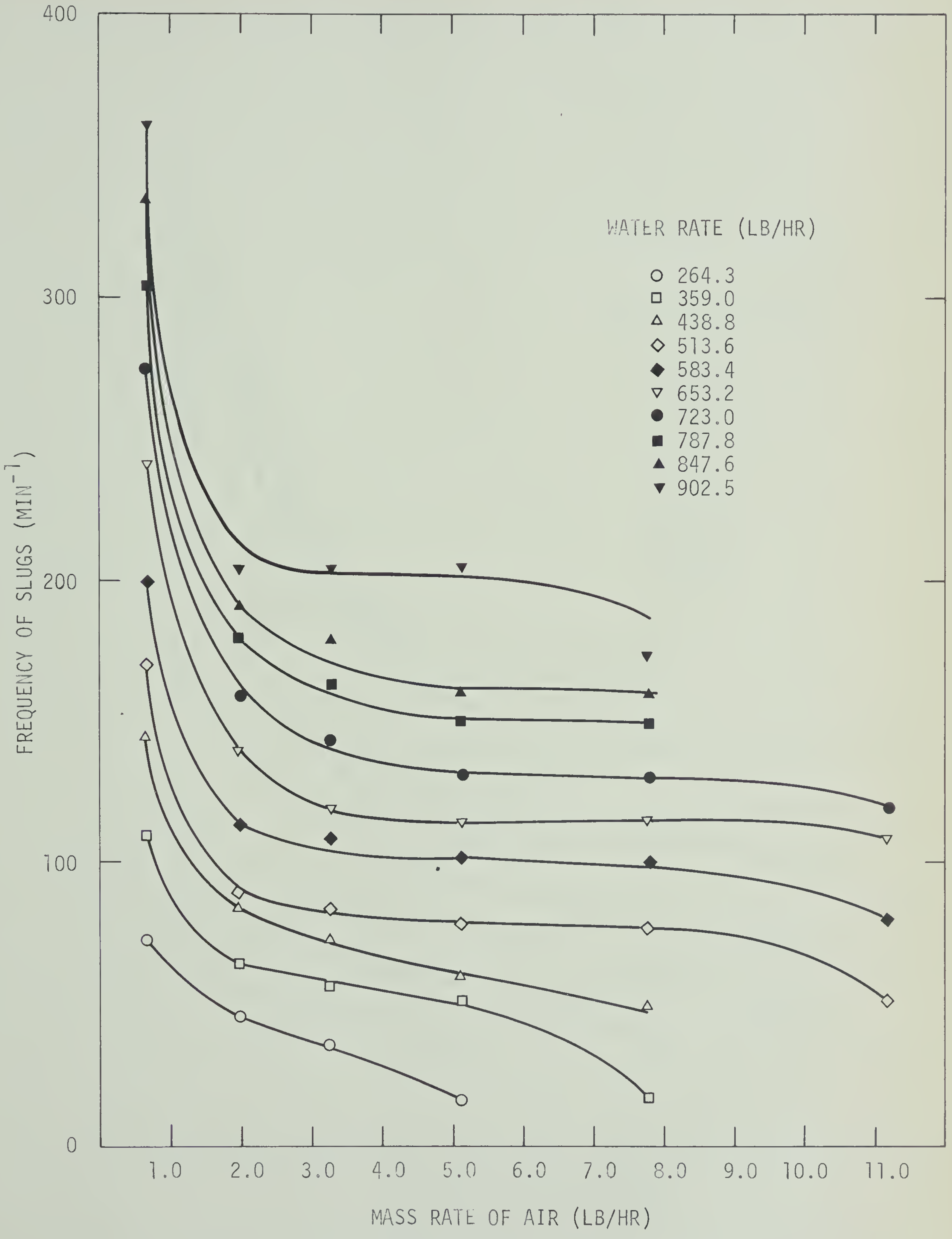


FIG. 19 - SLUG FREQUENCY FOR HORIZONTAL FLOW

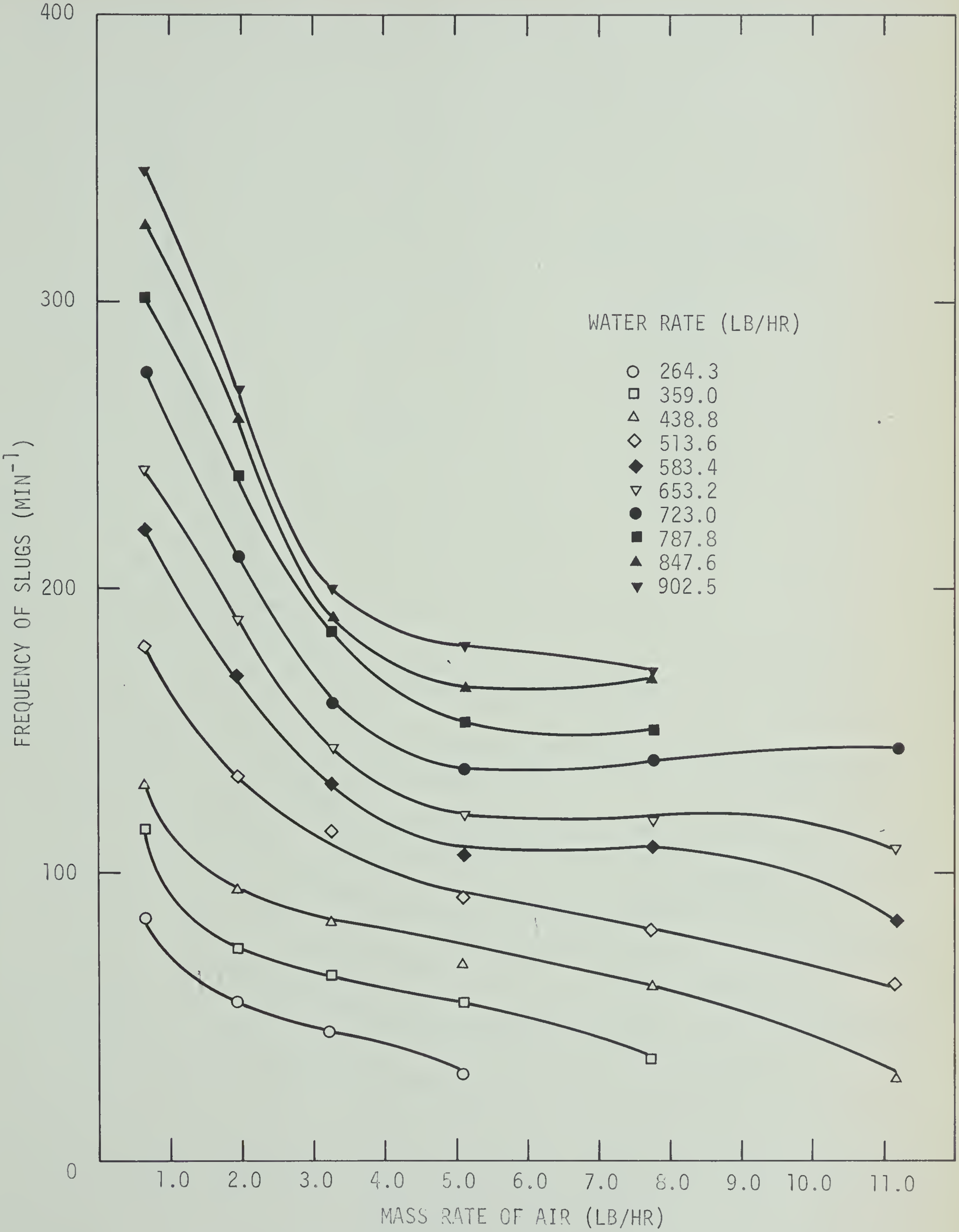


FIG. 20 - SLUG FREQUENCY FOR POSITIVELY SLOPED FLOW (+7°)

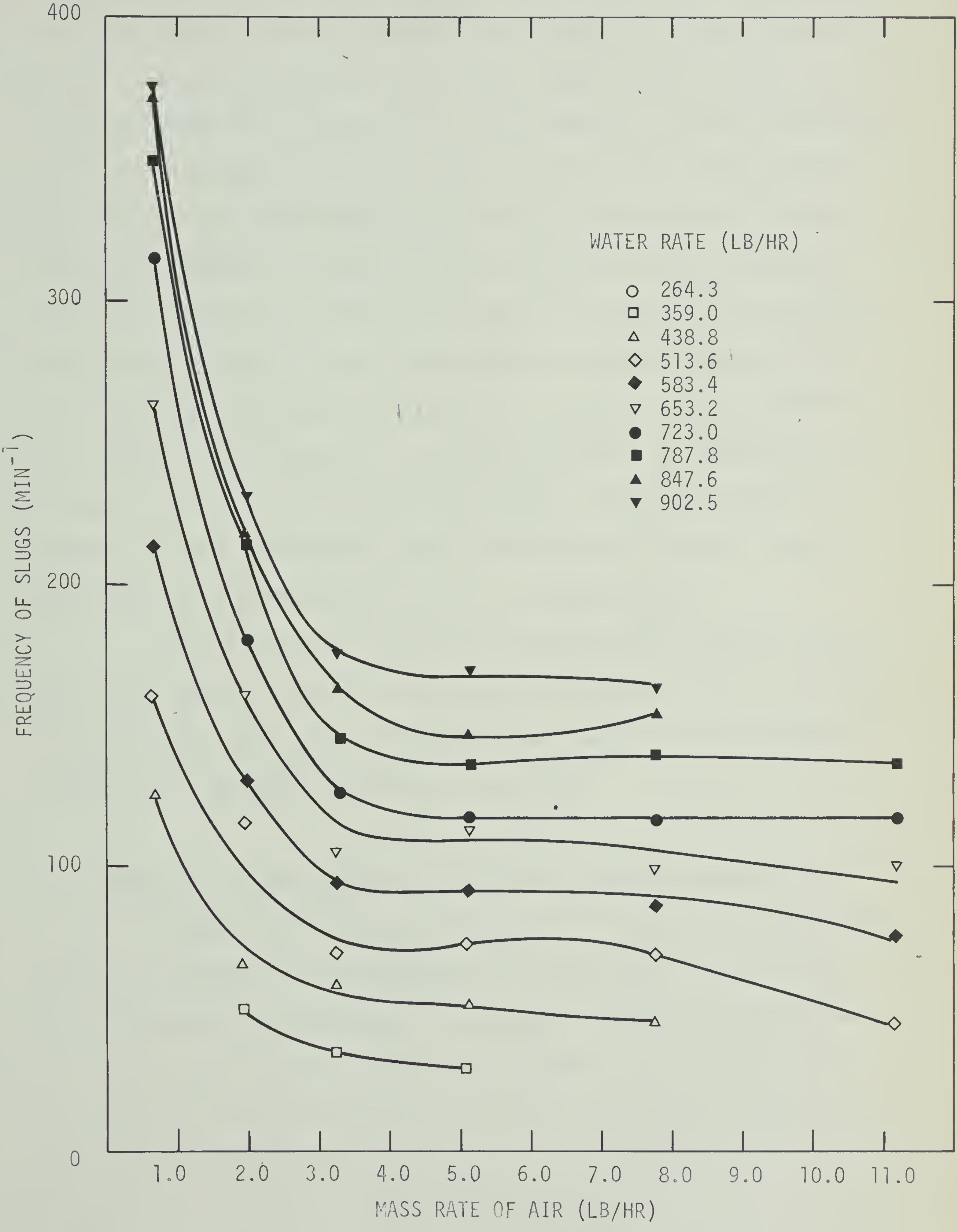


FIG. 21 - SLUG FREQUENCY FOR NEGATIVELY SLOPED FLOW (-7°)

The change in slope in the constant water rate curves is believed to be due to a flow pattern transition. It appears that the change in slope represents the transition from plug flow to slug flow since it coincides very closely with the change from the flow pattern of clean, froth-free, plugs to one in which there is an ever increasing amount of froth within the slug. The possibility that this phenomenon is a function of entrance conditions should, however, not be ruled out. Kordyban's (28) work, although carried out in a much lower region of slug flow, does show a similar occurrence for his data. At his maximum water rate the shape of the curve, plotted on the same axes as in this work, is almost identical with those of this work. It could therefore be concluded that the effect of entrance conditions is negligible with respect to the shape of the frequency curves, even though Kordyban's entrance section consisted of a ten inch section of vertical tubing, while that of this work had 17.9 ft. of straight tubing.

The only effect of tube inclination on slug frequency appears to be the location of the discontinuity. In horizontal flow the change is much more gradual than for the other two slopes, with not much difference being apparent between the latter two.

6.3 Comparison of the Pressure Drop Correlations with the Data

Figures 22 to 27 show a comparison of the pressure drop correlations of Lockhart-Martinelli (30) and Kordyban (27), and the slug theory developed in this work, with the data. Although the data from all three tube inclinations have been presented, static head was not considered in the correlations. As a result, comparisons can only be made with the horizontal data.

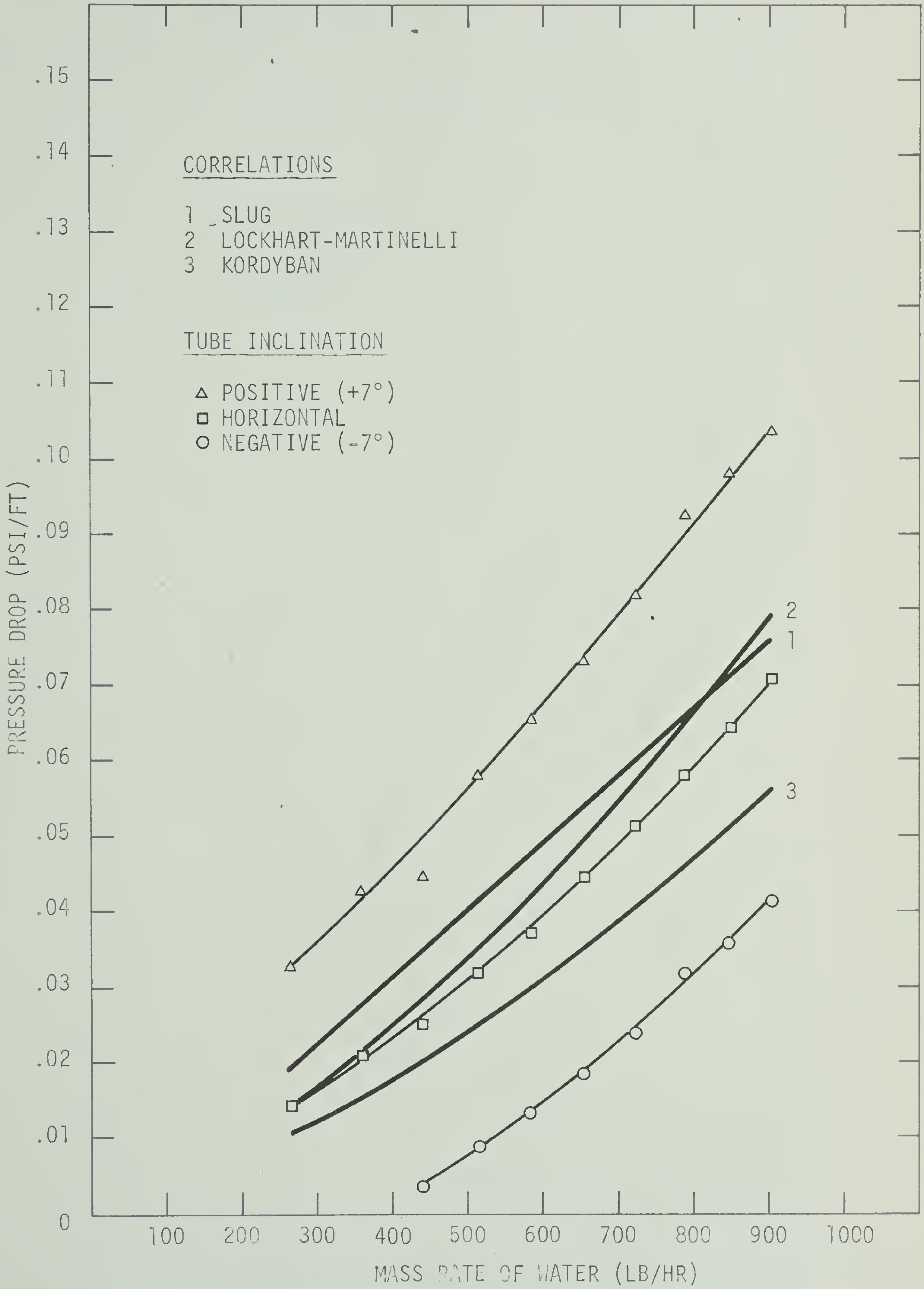


FIG. 22 - COMPARISON OF THE PRESSURE DROP CORRELATIONS WITH THE DATA AT 0.65 LB/HR AIR RATE

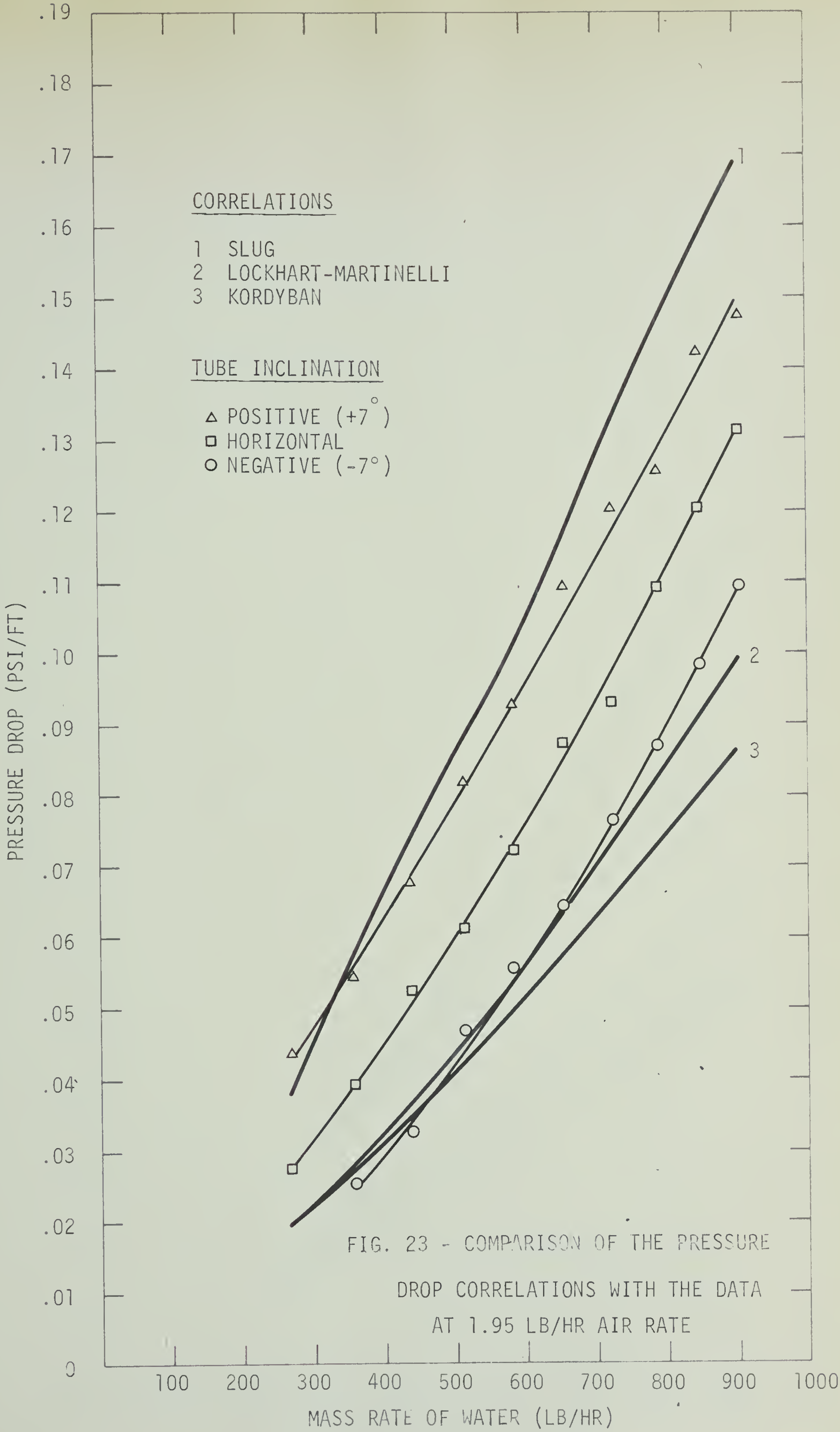


FIG. 23 - COMPARISON OF THE PRESSURE DROP CORRELATIONS WITH THE DATA AT 1.95 LB/HR AIR RATE

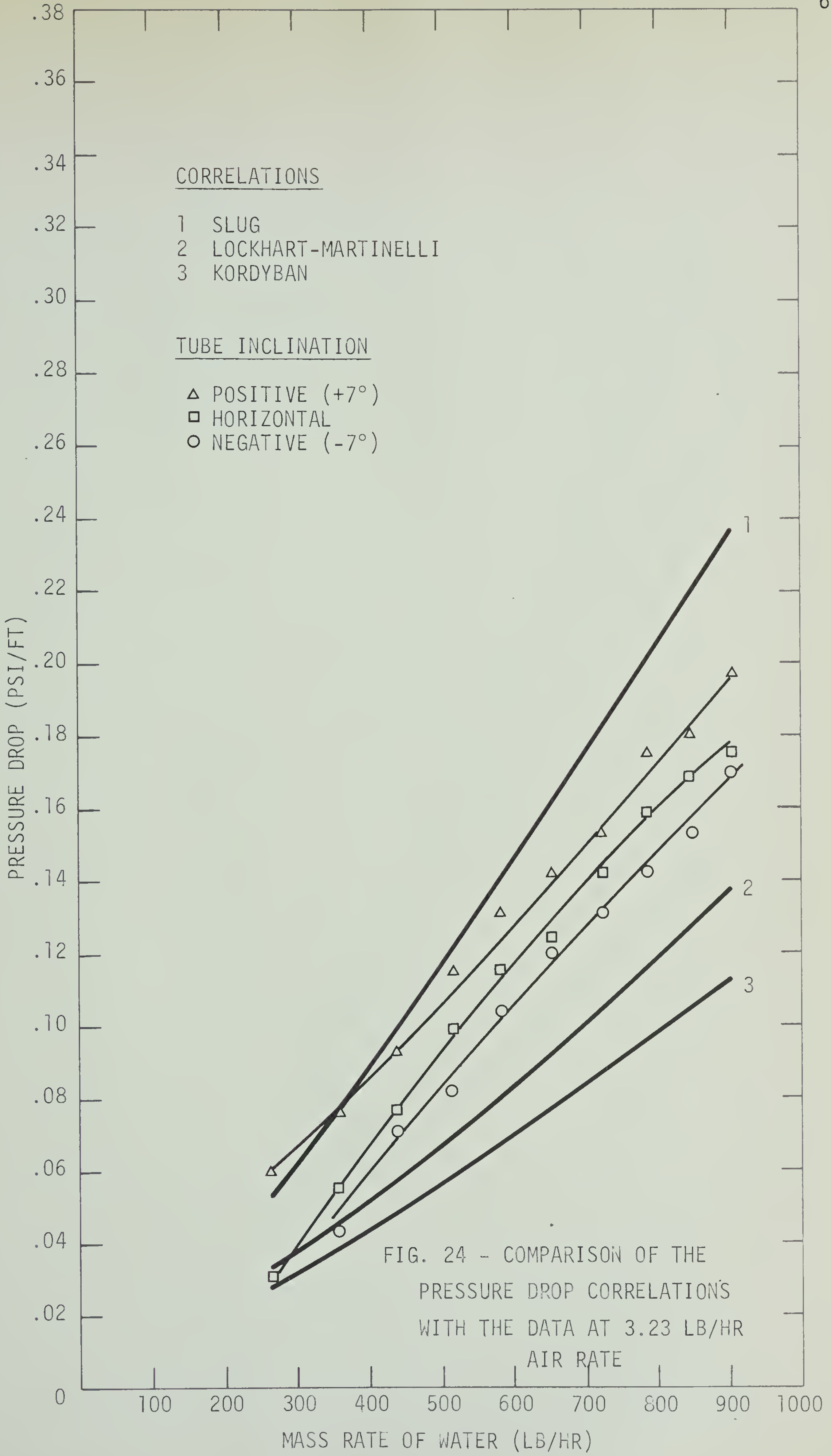


FIG. 24 - COMPARISON OF THE PRESSURE DROP CORRELATIONS WITH THE DATA AT 3.23 LB/HR AIR RATE

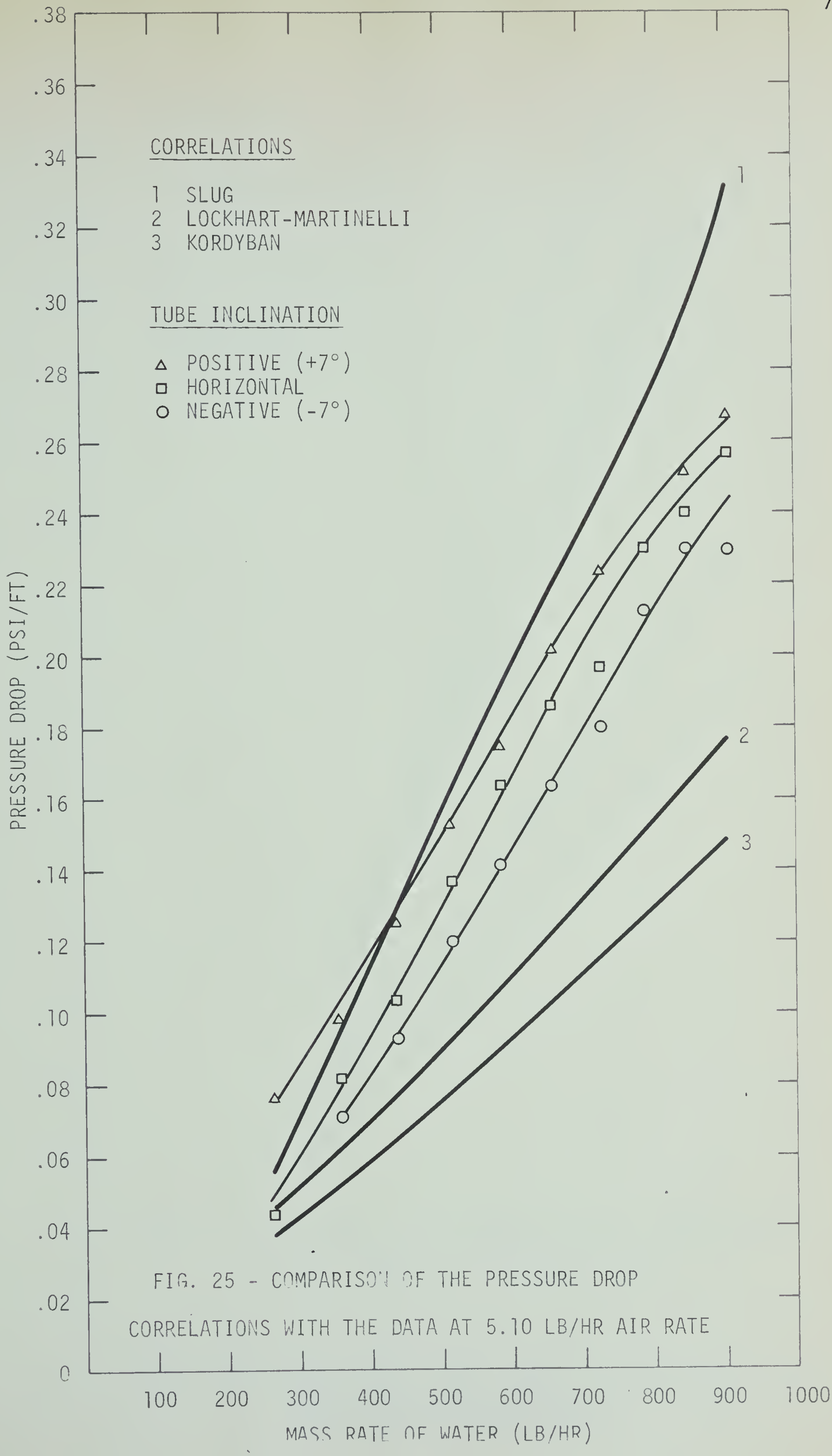


FIG. 25 - COMPARISON OF THE PRESSURE DROP CORRELATIONS WITH THE DATA AT 5.10 LB/HR AIR RATE

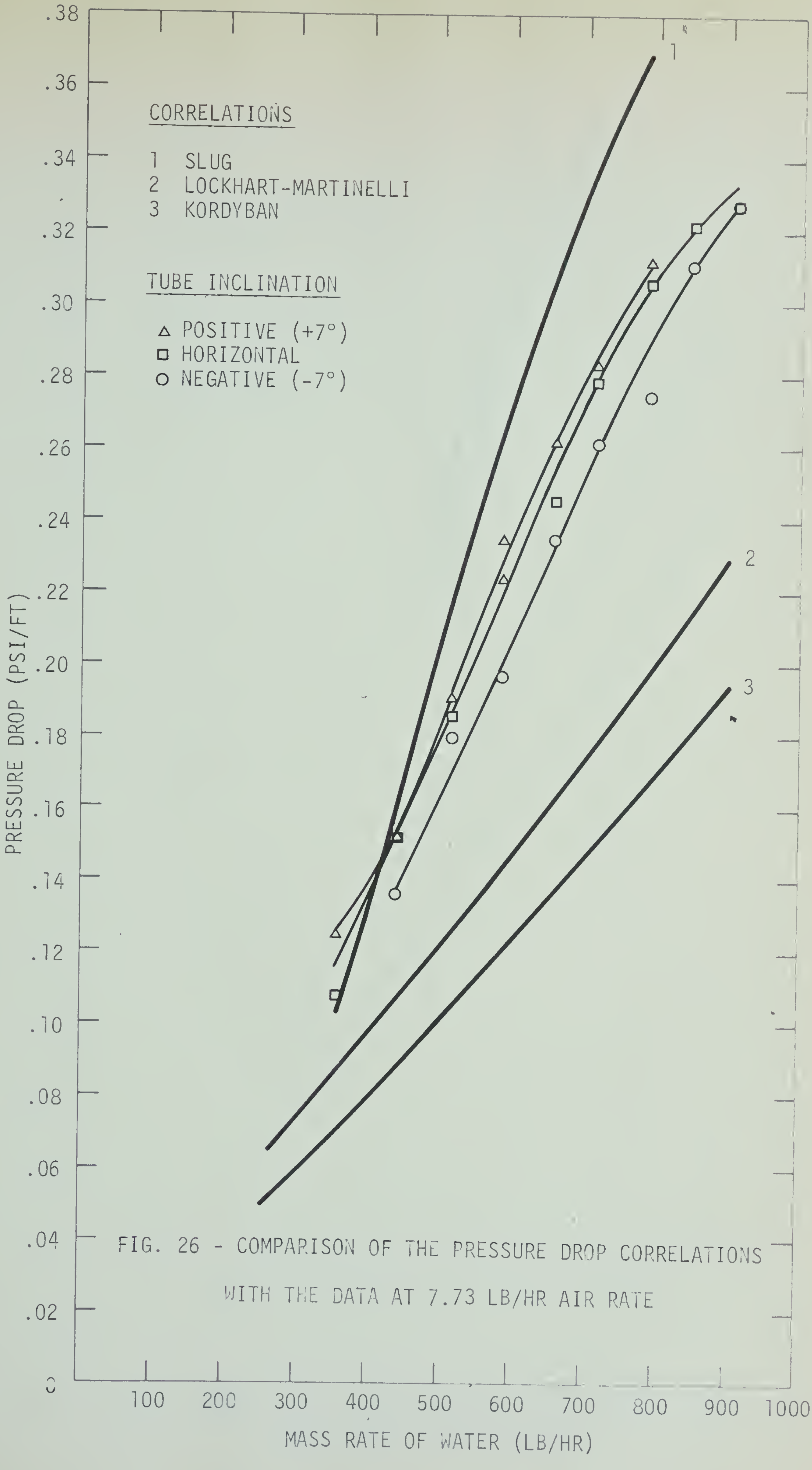


FIG. 26 - COMPARISON OF THE PRESSURE DROP CORRELATIONS WITH THE DATA AT 7.73 LB/HR AIR RATE

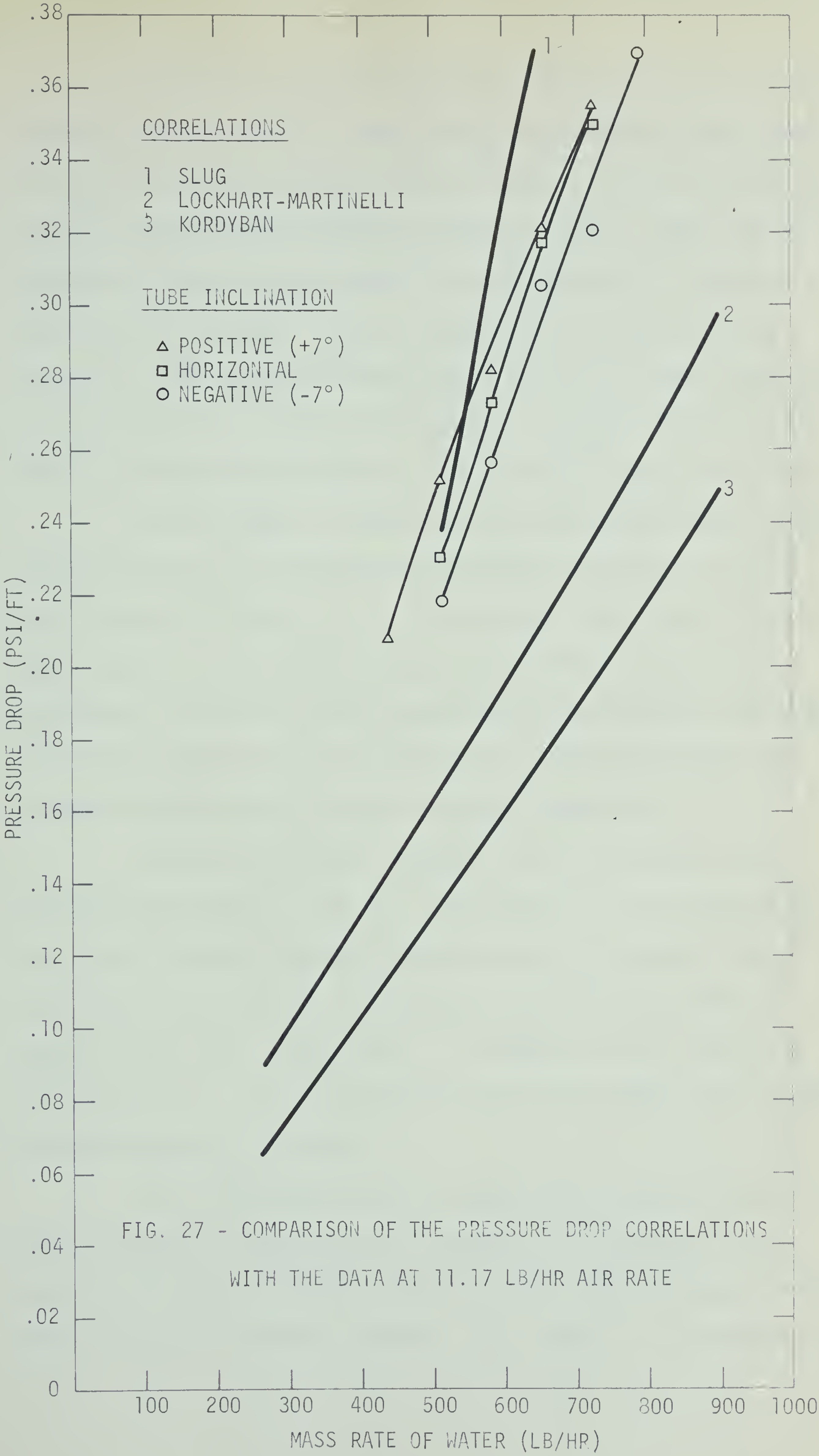


FIG. 27 - COMPARISON OF THE PRESSURE DROP CORRELATIONS WITH THE DATA AT 11.17 LB/HR AIR RATE

In comparing the predictions of Lockhart-Martinelli and of Kordyban with the data it is seen that good agreement has only been obtained for the case of 0.65 lb/hr air rate. As the air rate is increased, the deviations increase markedly. The data range from 30 to 50 percent higher than predicted by Lockhart-Martinelli, and even higher than by Kordyban. Similar observations have been made by Brigham (7), who reported pressure drops five to six times those predicted with flow inclined at 12.4 degrees. Kordyban (27) also found that his predictions were some 20 to 30 percent higher than his data.

The slug model, on the other hand, shows much better agreement with the data. Although the predictions increase too rapidly at the lower air rates, i.e., up to 40 percent higher than the data, they return to within 25 to 30 percent above the data over most of the range. The fact that the predictions are consistently high makes this model superior to the correlations of Lockhart-Martinelli and Kordyban for slug flow since it provides an upper bound.

It must be admitted, however, that the slug model only approximates the actual slug flow. Such terms as slug acceleration, would tend to become important with the pressure gradients that were encountered in this work. Even pressure drop in the gas phase, momentum loss in the liquid slugs, and others of the simplifying assumptions made in the development of the film slowdown problem could become increasingly important.

It is unfortunate that the model could not be derived from purely mechanistic considerations. There is, at present, no such method available for predicting film thickness, slug frequency and the number of slugs in the test section. As a result it was necessary to

employ actual data, or to make approximations using the data, to be able to solve the problem. Nevertheless, the model's predictions are far more reliable and useful, under the conditions studied, than the predictions of Lockhart-Martinelli and Kordyban.

CHAPTER VII

CONCLUSIONS

1. A region of intermittent slug flow, which exists between stratified or wavy flow and slug flow, is strongly influenced by tube inclination.

2. In the transition from wavy or stratified flow to intermittent slug flow it is possible to induce slug formation by artifically creating pressure fluctuations.

3. The region, on a Baker plot, of regular slug flow is not influenced by a tube inclination of $\pm 7°$. This indicates a stable flow pattern for slug flow.

4. Pressure gradient, pressure fluctuations, slug velocity, and slug frequency are more strongly influenced by air rate than by the water rate, the other remaining constant.

5. Tube inclination, within $\pm 7°$, does not have a significant influence on pressure gradient (neglecting static head), pressure fluctuations, slug velocity, and slug frequency. This also shows slug flow to be a stable flow pattern.

6. A sharp reduction in the slug frequency occurs at the transition between plug and slug flow.

7. Slug velocity and pressure gradient are influenced by the transition from plug flow, through slug flow, to semi-annular flow.

8. Pressure fluctuations in slug flow attain a maximum before declining at the onset of semi-annular flow.

9. The observed slug velocity is up to 50 percent higher than the calculated total fluid velocity.

10. The Lockhart-Martinelli and Kordyban correlations predict pressure drop 30 to 50 percent lower than the slug flow data.

11. The pressure drop predictions from the proposed slug flow model are 25 to 30 percent higher than the slug flow data. This provides an upper bound for slug flow pressure drop predictions.

NOMENCLATURE

A	tube cross sectional area
D	tube diameter
f	Fanning friction factor
F	force
g	gravitational acceleration, gravitational conversion factor
G	mass velocity of gas
k	thermal conductivity
ℓ	film thickness
L	length, mass velocity of liquid
N	number
P	pressure
Q	volumetric flow rate
R	holdup
Re	Reynolds number
r	radius
S	film cross sectional area
T	temperature
t	time
u	velocity
w	mass flow rate
X	$[(\frac{\Delta P}{\Delta L})_L/(\frac{\Delta P}{\Delta L})_G]^{1/2}$ Lockhart-Martinelli parameter

Greek Letters

Δ	difference
ε	liquid holdup in film

λ	$[(\rho_G/0.075)(\rho_L/62.3)]^{1/2}$ = parameter of Baker area plot for two-phase flow
μ	viscosity
ν	kinematic viscosity = μ/ρ
π	3.14159 ...
ρ	density
σ	surface tension
τ	shear stress
Φ	$[(\frac{\Delta P}{\Delta L})_{TP}/(\frac{\Delta P}{\Delta L})_L]^{1/2}$ Lockhart-Martinelli parameter
ψ	$(73/\sigma)\ [\mu_L(62.2/\rho_L)^2]^{1/3}$ = parameter of Baker area plot for two-phase flow

Subscripts

c	constant
f	from friction considerations
G	gas
L	liquid
m	from momentum considerations
s	slug flow
TP	two-phase
t	test
TOT	total
1	initial
2	final

BIBLIOGRAPHY

1. Alves, G.F., Chem. Eng. Progr, 50 No. 9, 449 (1954).
2. Anderson, R.J., and Russell, T.W.F., Chem. Eng., 72 Pt 4, 99 (1965).
3. Anderson, R.J., and Russell, T.W.F., Chem. Eng., 72 Pt 4, 139 (1965).
4. Baker, O., Oil and Gas J. 53 No. 12, 185 (1954).
5. Bankoff, S.G., Trans. Am. Soc. Mech. Engrs., 82 265 (1960).
6. Boelter, L.M.K., and Kepner, R.H., Ind. Eng. Chem. 31, 426 (1939).
7. Brigham, W.E., Holstein, E.D., and Huntington, R.L., Pet. Engr., 29 No. 3, D39 (1957).
8. Brodkey, R.S., The Phenomena of Fluid Motions, Chap. 16, Addison-Wesley, Reading, Mass. (1967).
9. Brown, G.G., and Associates, Unit Operations, Wiley, New York, 545, (1956).
10. Calvert, S., Ph.D. Thesis, University of Michigan (1952).
11. Chenoweth, J.M., and Martin, M.W., Pet. Ref. 34, No. 10, 151 (1955).
12. Chisholm, D., A Review of the Literature on Two-Phase Flow, 1931 to 1962, National Engineering Laboratory, Scotland (1963).
13. Chisholm, D., Instn. of Mech. Engrs. (England), 8 No. 1, 107 (1966).
14. Churchill, R.V., Modern Operational Mathematics in Engineering, McGraw-Hill, New York, 196 (1944).
15. Cromer, S., and Huntington, R.L., Trans. A.I.M.E., Petr. Dev. and Tech. 136, 79 (1940).

16. Davis, G.I.,Jr., and Weidner, C.R., Univ. of Wisconsin, Bulletin 450, (1911).

17. Dukler, A.E., Wicks, M., III, and Cleveland, R.G., A.I.Ch.E. J. 10, 38 (1964).

18. Dukler, A.E., Wicks, M., III, and Cleveland, R.G., A.I.Ch.E. J. 10, 44 (1964).

19. Dukler, A.E., and Wicks, M., III, Modern Chemical Engineering (A. Acrivos, Editor), Chap. 8, Reinhold, New York (1963).

20. Dukler, A.E., American Pipeline Institute 16th Annual Pipeline Conference, 80 (1965).

21. Flanigan, O., Oil and Gas J., 132 (1958).

22. Galegar, W.C., Stovall, W.C., and Huntington, R.L., Pet. Ref. (1954).

23. Gouse, S.W., Jr., Index to the Two-Phase Gas-Liquid Flow Literature, MIT Rept. No. 9, The MIT Press, Cambridge, Mass. (1966).

24. Hoogendoorn, C.J., Chem. Eng. Sci. 9, 205 (1959).

25. Hughmark, G.A., Chem. Eng. Sci., 20, 1007 (1965).

26. Kepple, R.R., and Tung, T.V., ANL-6734, Argonne National Laboratory, Argonne, Ill. (1963).

27. Kordyban, E.S., Trans. A.S.M.E., Ser. D., 83, No. 4, 613 (1961).

28. Kordyban, E.S., and Ranov, T., A.S.M.E., Hydr. and Heat Transf. Div., Multiphase Flow Symposium, 1 (1963).

29. Kosterin, S.I., Izvest. Akad. Nauk SSSR Otdel. Tekh. Nauk, 1824 (1949).

30. Lockhart, R.W., and Martinelli, R.C., Chem. Eng. Progr. 45, 39 (1949).

31. Lorenz, H., Zeitschrift des Vereines Deutscher Ingenieure, 53, 545 (1909).

32. McDonald, A.E., and Baker, O., Paper presented at the Mid-Continent District API, Division of Production, May (1964).

33. McManus, H.N., Jr., A.S.M.E., Hydr. and Heat Transf. Div., Multiphase Flow Symposium, 75 (1963).

34. Martinelli, R.C., Boelter, L.M., Taylor, T.H.M., Thomsen, E.G., and Morrin, E.A., Trans. A.S.M.E. 66 139 (1944).

35. Moore, T.V., and Wilde, H.D., Jr., Trans A.I.M.E., Petr. Dev. and Tech., 92, 296 (1931).

36. Neal, L.G., and Bankoff, S.G., A.I.Ch.E. J., 9, 490 (1963).

37. Nowels, K.B., Trans. A.I.M.E., Petr. Dev. and Tech. 98, 401 (1932).

38. Petrick, M., ANL-5787, Argonne National Lab, Argonne, Ill. (1958).

39. Poettman, F.H., and Carpenter, P.G., Paper No. 851-26-J, Mid-Continent District API Division of Production (1952).

40. Ryan, J.T., and Vermeulen, L.R., A Slug Flow Transducer, Review of Scientific Instruments, (in print).

41. Scott, D.S., Advances in Chemical Engineering (Drew, T.B., and others Ed.) 4, Academic Press, New York, 199 (1963).

42. Uren, L.C., Gregory, P.P., Hancock, R.A., and Feskow, G.W., Trans. A.I.M.E. Petr. Dev. and Tech., 86, 209 (1930).

43. White, P.D., and Huntington, R.L., The Pet. Eng., 27, 615(1955).

44. Wood, R.K., M.Sc. Thesis in Chemical Engineering, Univ. of Alberta (1960).

45. Yagi, S., Chem. Eng. (Japan) 18, 2 (1954).

APPENDIX - A

(1) AUXILIARY DATA

(2) EXPERIMENTAL DATA

TABLE A-1

AUXILIARY DATA

PIPELINE MATERIAL - CLEAR EXTRUDED LUCITE

Tube Diameter	= 0.0417 feet
Test Section Length	= 18.29 feet
Entrance Length	= 17.9 feet
Exit Length	= 4.0 feet
Angles of Inclination	= +7°0', 0°0', -7°0'

PHYSICAL PROPERTIES OF FLUIDS

(At 80°F, 1 ATM)

	WATER	AIR (SATURATED WITH WATER)
Viscosity	0.900 CP	0.018 CP
Density	62.20 lb/ft^3	0.0708 lb/ft^3

Surface Tension 67.2 Dynes/Cm

TABLE A-2

EXPERIMENTAL DATA

HORIZONTAL FLOW

RATES			SLUG		PRESSURES	
AIR (CFM)	WATER (USGPM)	FREQ (MIN^{-1})	TIME (SEC.)	VEL. (FT/SEC)	ΔP (PSI)	FLUCT. (PSI)
.153	.53	72	5.5	3.33	.26	.12
	.72	110	5.0	3.65	.38	.08
	.88	145	4.5	4.06	.46	.08
	1.03	171	4.3	4.25	.58	.08
	1.17	201	4.5	4.06	.68	.06
	1.31	242	4.2	4.35	.82	.06
	1.45	276	3.7	4.95	.94	.08
	1.58	305	3.7	4.95	1.06	.08
	1.70	335	3.2	5.70	1.18	.08
	1.81	361	3.5	5.23	1.30	.08
.460	.53	41	2.5	7.31	.50	.40
	.72	65	2.2	8.32	.72	.50
	.88	84	2.2	8.32	.96	.44
	1.03	90	2.0	9.15	1.12	.46
	1.17	114	2.0	9.15	1.32	.50
	1.31	140	2.0	9.15	1.60	.50
	1.45	160	1.9	9.64	1.70	.50
	1.58	180	1.8	10.2	2.00	.44
	1.70	192	1.7	10.8	2.20	.44
	1.81	205	1.7	10.8	2.40	.50
.760	.53	36	1.7	10.8	.56	1.20
	.72	57	1.6	11.4	1.00	1.20
	.88	74	1.5	12.2	1.40	1.20
	1.03	85	1.4	13.1	1.80	1.30
	1.17	108	1.3	14.0	2.10	1.20
	1.31	120	1.2	15.2	2.30	1.10
	1.45	144	1.1	16.6	2.60	1.10
	1.58	168	1.1	16.6	2.90	1.00
	1.70	180	1.1	16.6	3.10	.90
	1.81	205	1.1	16.6	3.20	1.00

EXPERIMENTAL DATA

HORIZONTAL FLOW

RATES			SLUG		PRESSURES	
AIR (CFM)	WATER (USGPM)	FREQ (MIN^{-1})	TIME (SEC.)	VEL. (FT/SEC)	ΔP (PSI)	FLUCT. (PSI)
1.20	.53	16	.95	19.3	.80	1.20
	.72	52	.90	20.4	1.50	1.60
	.88	60	.88	20.8	1.90	2.00
	1.03	79	.84	21.8	2.50	2.10
	1.17	102	.84	21.8	3.00	2.40
	1.31	115	.80	22.9	3.40	2.50
	1.45	132	.76	24.1	3.60	2.60
	1.58	152	.76	24.1	4.20	2.60
	1.70	162	.74	24.7	4.40	2.20
	1.81	206	.74	24.7	4.70	2.00
1.82	.53	-	-	-	-	-
	.72	18	.60	30.5	2.00	2.00
	.88	50	.60	30.5	2.80	2.20
	1.03	77	.55	33.3	3.40	2.50
	1.17	100	.50	36.6	4.10	2.70
	1.31	115	.50	36.6	4.50	2.90
	1.45	132	.47	39.0	5.10	3.00
	1.58	150	.47	39.0	5.60	3.20
	1.70	160	.46	39.7	5.90	3.20
	1.81	174	.44	41.5	6.00	3.00
2.63	.53	-	-	-	-	-
	.72	-	-	-	-	-
	.88	-	-	-	-	-
	1.03	52	.43	42.5	4.20	2.40
	1.17	80	.38	48.1	5.00	2.60
	1.31	108	.35	52.2	5.80	2.80
	1.45	120	.35	52.2	6.40	3.00
	1.58	-	-	-	-	-
	1.70	-	-	-	-	-
	1.81	-	-	-	-	-

EXPERIMENTAL DATA

POSITIVELY INCLINED FLOW (+7°)

RATES			SLUG		PRESSURES	
AIR (CFM)	WATER (USGPM)	FREQ (MIN^{-1})	TIME (SEC.)	VEL. (FT/SEC)	ΔP (PSI)	FLUCT. (PSI)
0.153	.53	84	6.0	3.05	.60	.19
	.72	116	4.8	3.81	.78	.14
	.88	132	4.4	4.15	.82	.14
	1.03	180	4.1	4.46	1.06	.12
	1.17	220	4.1	4.46	1.20	.10
	1.31	240	3.8	4.81	1.34	.10
	1.45	276	3.5	5.22	1.50	.09
	1.58	302	3.5	5.22	1.70	.10
	1.70	327	3.3	5.55	1.80	.10
	1.81	346	3.3	5.55	1.90	.10
.460	.53	56	2.4	7.63	.80	.54
	.72	74	2.3	7.95	1.00	.52
	.88	95	1.9	9.64	1.24	.52
	1.03	134	1.8	10.2	1.50	.50
	1.17	168	1.9	9.64	1.70	.50
	1.31	188	1.8	10.2	2.00	.50
	1.45	212	1.8	10.2	2.20	.50
	1.58	240	1.7	10.7	2.30	.50
	1.70	260	1.65	11.1	2.60	.50
	1.81	270	1.6	11.1	2.70	.50
.760	.53	45	1.5	12.2	1.10	1.10
	.72	65	1.4	13.1	1.40	1.20
	.88	84	1.35	13.5	1.70	1.20
	1.03	115	1.30	14.0	2.10	1.20
	1.17	132	1.20	15.2	2.40	1.20
	1.31	144	1.15	15.9	2.60	1.20
	1.45	160	1.05	17.4	2.80	1.20
	1.58	185	1.05	17.4	3.20	1.20
	1.70	190	1.05	17.4	3.30	1.10
	1.81	200	1.05	17.4	3.60	1.10

[illegible]

EXPERIMENTAL DATA

POSITIVELY INCLINED FLOW (+7°)

RATES AIR (CFM)	RATES WATER (USGPM)	FREQ (MIN^{-1})	SLUG TIME (SEC.)	VEL. (FT/SEC)	PRESSURES ΔP (PSI)	PRESSURES FLUCT. (PSI)
1.20	.53	30	1.0	18.3	1.40	1.50
	.72	55	.90	20.4	1.80	1.80
	.88	68	.80	22.9	2.30	2.00
	1.03	92	.78	23.5	2.80	2.22
	1.17	107	.75	24.4	3.20	2.40
	1.31	122	.75	24.4	3.90	2.50
	1.45	137	.75	24.4	4.10	2.50
	1.58	153	.73	25.1	4.20	2.20
	1.70	166	.75	25.1	4.60	2.00
	1.81	180	.74	24.7	4.90	1.80
1.82	.53	-	-	-	-	-
	.72	36	.62	29.6	2.30	1.80
	.88	61	.61	30.0	2.80	2.40
	1.03	80	.58	31.6	3.50	2.60
	1.17	110	.55	33.3	4.30	2.70
	1.31	118	.50	36.6	4.80	3.00
	1.45	140	.50	36.6	5.20	3.00
	1.58	151	.50	36.6	5.70	2.80
	1.70	168	.48	37.4	-	-
	1.81	171	.47	36.6	-	-
2.63	.53	-	-	-	-	-
	.72	-	-	-	-	-
	.88	28	.44	41.6	3.80	2.20
	1.03	62	.45	40.7	4.60	2.60
	1.17	84	.41	44.6	5.20	2.80
	1.31	109	.38	48.2	5.90	2.80
	1.45	144	.36	50.8	6.50	2.70
	1.58	-	-	-	-	-
	1.70	-	-	-	-	-
	1.81	-	-	-	-	-

EXPERIMENTAL DATA

NEGATIVELY INCLINED FLOW (-7°)

RATES AIR (CFM)	RATES WATER (USGPM)	FREQ (MIN^{-1})	SLUG TIME (SEC.)	VEL. (FT/SEC)	PRESSURES ΔP (PSI)	PRESSURES FLUCT (PSI)
.153	.53	-	-	-	-	-
	.72	-	-	-	-	-
	.88	126	-	-	.06	.02
	1.03	160	-	-	.15	.03
	1.17	214	-	-	.24	.03
	1.31	264	-	-	.33	.04
	1.45	316	-	-	.43	.04
	1.58	350	-	-	.57	.05
	1.70	372	-	-	.65	.05
	1.81	376	-	-	.76	.06
.460	.53	-	-	-	-	-
	.72	50	2.20	8.30	.46	.28
	.88	66	2.00	9.15	.60	.30
	1.03	115	1.92	9.52	.86	.32
	1.17	131	1.85	9.90	1.02	.38
	1.31	161	1.80	10.15	1.18	.40
	1.45	181	1.75	10.4	1.40	.40
	1.58	214	1.73	10.6	1.60	.40
	1.70	218	1.65	11.1	1.80	.40
	1.81	232	1.63	11.2	2.00	.40
.760	.53	10	1.3	14.1	-	-
	.72	35	1.2	15.2	.80	.80
	.88	58	1.16	15.8	1.30	1.00
	1.03	69	1.10	16.6	1.50	1.20
	1.17	95	1.10	16.6	1.90	1.20
	1.31	105	1.14	16.0	2.20	1.10
	1.45	127	1.15	15.9	2.40	1.00
	1.58	141	1.10	16.6	2.60	.90
	1.70	163	1.10	16.6	2.80	.90
	1.81	175	1.07	17.1	3.10	.80

APPENDIX - B

(1) COMPUTER PROGRAM

(2) CALCULATED DATA

```
C
C      THE PURPOSE OF THIS PROGRAM IS TO MAKE PREDICTIONS
C      OF PRESSURE DROP FOR HORIZONTAL SLUG FLOW BY VARIOUS
C      PRESSURE DROP CORRELATIONS
C
C      THE CORRELATIONS ARE,
C      1. LOCKHART-MARTINELLI
C      2. KORDYBAN
C      3. SLUG FLOW MODEL
C
C      MAINLINE PROGRAM AND
C      LOCKHART-MARTINELLI CORRELATION FOR TWO-PHASE FLOW
C
       COMMON VISCG,VISCL,DIA,PI,GEE,DENSL,DENSG,RELP,REGP,
      1 KORFUN,DPL,DPKOR ,COEFFL,COEFFG,PHIL,PHIG,RL,N,M,
      2 RATL(25),RATG(25),I,J,DPSLUG(60),FREQH(60),FREQP(60),
      3 FREQN(60),VELH(60),VELP(60),VELN(60),PRESSH(60),
      4 PRESSP(60),PRESSN(60),K,KK
       LOGICAL TL,TG,VL,VG
       REAL M,N,KORFUN
C
C      PHYSICAL PROPERTIES AND COUNTERS ARE READ IN
C      VISCOSITIES IN CENTIPOISE
C      DENSITIES IN POUNDS PER CUBIC FOOT
C      TUBE DIAMETER IN FEET
C
       READ(5,200) VISCG,VISCL,DENSG,DENSL,DIA
       READ(5,203) KK,MM,NN
C
C      SLUG FREQUENCIES, VELOCITIES, AND PRESSURE DROPS FOR
C      HORIZONTAL AND POSITIVE AND NEGATIVE SLOPES ARE READ IN
C      THE LAST LETTER BEFORE THE SUBSCRIPT REFERS TO THE
C      TUBE INCLINATION
C      H  HORIZONTAL
C      P  POSITIVE
C      N  NEGATIVE
C
       READ(5,204)((FREQH(K),VELH(K),PRESSH(K)),K=1,KK)
       READ(5,204)((FREQP(K),VELP(K),PRESSP(K)),K=1,KK)
       READ(5,204)((FREQN(K),VELN(K),PRESSN(K)),K=1,KK)
C
C      RATES ARE READ IN
C      WATER IN U.S. GPM
C      AIR IN CFM
C
       READ (5,201) (RATL(J),J=1,MM)
       READ (5,202) (RATG(I),I=1,NN)
```

```
      PI=3.14159
      GEE=32.17
C
C     HEADINGS FOR THE CALCULATED DATA ARE PRINTED
C
      WRITE(6,1000)
      WRITE(6,105)
      WRITE(6,102)
      WRITE(6,104)
C
C     RATES CONVERTED TO LBS PER HOUR
C
      DO 3 J=1,MM
    3 RATL(J)=RATL(J)*60.0*.1336*DENSL
      DO 2 I=1,NN
    2 RATG(I)=RATG(I)*60.0*DENSG
      K=1
      DO 1 I=1,NN
      DO 1 J=1,MM
      TL=.FALSE.
      TG=.FALSE.
      VL=.FALSE.
      VG=.FALSE.
C
C     THE REYNOLDS NUMBERS FOR THE GAS AND LIQUID PHASES
C     ARE CALCULATED, ASSUMING EACH PHASE TO FLOW ALONE
C
      REGP=(4.0*RATG(I))/(VISCG*DIA*PI*2.4191)
      RELP=(4.0*RATL(J))/(VISCL*DIA*PI*2.4191)
C
C     DETERMINATION WHETHER EACH OF THE PHASES IS IN EITHER
C     TURBULENT OR LAMINAR(VISCOUS) FLOW
C
      IF(REGP.LE.2000.0) VG=.TRUE.
      IF(RELP.LE.2000.0) VL=.TRUE.
      IF(RELP.GT.2000.0) TL=.TRUE.
      IF(REGP.GT.2000.0) TG=.TRUE.
C
C     DEPENDING ON THE CHOICE OF FLOW MECHANISM,ONE OF THE
C     FOUR SUBROUTINES IS CALLED TO CALCULATE THE PARAMETERS
C
      IF(TL.AND.TG) CALL TURTUR
      IF(TL.AND.VG) CALL TURVIS
      IF(VL.AND.TG) CALL VISTUR
      IF(VL.AND.VG) CALL VISVIS
C
C     FRICTION FACTORS ARE CALCULATED BY THE BLASIUS EQUATION
```

```
C
      FFG=COEFFG/(REGP**M)
      FFL=COEFFL/(RELP**N)
C
C     SINGLE PHASE PRESSURE DROP CALCULATIONS
C     BY FANNING EQUATION
C
      DPL=(32.0*FFL*(RATL(J)**2))/(DENSL*(DIA**5)*(PI**2)
     1 *GEE*(3600.0**2)*144.0)
      DPG=(32.0*FFG*(RATG(I)**2))/(DENSG*(DIA**5)*(PI**2)
     1 *GEE*(3600.0**2)*144.0)
C
C     TWO-PHASE PRESSURE DROP CALCULATIONS FOR LOCKHART-
C     MARTINELLI CORRELATION
C
      DPTPL=(PHIL**2)*DPL
      DPTPG=(PHIG**2)*DPG
C
C     THE KORDYBAN CORRELATION IS CALLED
C
      CALL KORDY
C
C     THE EXPERIMENTAL DATA FOR OVERALL PRESSURE DROP
C     ARE CHANGED TO PRESSURE GRADIENT PSI/FT
C
      PRESSH(K)=PRESSH(K)/18.29
      PRESSP(K)=PRESSP(K)/18.29
      PRESSN(K)=PRESSN(K)/18.29
C
C     THE SLUG MODEL IS CALLED
C
      CALL SLUG
      IF((I.EQ.4).AND.(J.EQ.1)) GO TO 4
      GO TO 5
    4 WRITE(6,1000)
      WRITE(6,104)
    5 CONTINUE
      WRITE(6,103) RATL(J),RATG(I),DPTPL,DPKOR,DPSLUG(K),
     1 PRESSH(K),PRESSP(K),PRESSN(K)
      K=K+1
      LK=KK+1
      IF(K.EQ.LK) GO TO 10
    1 CONTINUE
  102 FORMAT(34X,'CALCULATED DATA'//)
  103 FORMAT(2X,F8.3,F10.3,6F10.5)
  104 FORMAT(5X,'RATES (LB/HR)',15X,'PRESSURE GRADIENT (PSI',
     1'/FT)'/5X,'WATER       AIR',6X,'L-M       KORDY      SLU',
```

```
     2'G        HORIZ        POS         NEG'/)
 105 FORMAT(37X,'TABLE B-1'/)
 200 FORMAT(5F10.5)
 201 FORMAT(4F15.5/4F15.5/2F15.5)
 202 FORMAT(4F15.5/2F15.5)
 203 FORMAT(3I3)
 204 FORMAT(3F10.2)
1000 FORMAT(1H1)
  10 STOP
     END
```

```
      SUBROUTINE VISVIS
C
C     SUBROUTINE VISCOUS-VISCOUS
C     CALCULATION OF THE PARAMETERS OF THE LOCKHART-MARTIN-
C     ELLI CORRELATION FOR LIQUID AND GAS BOTH IN VISCOUS
C     FLOW
C
      COMMON VISCG,VISCL,DIA,PI,GEE,DENSL,DENSG,RELP,REGP,
     1 KORFUN,DPL,DPKOR ,COEFFL,COEFFG,PHIL,PHIG,RL,N,M,
     2 RATL(25),RATG(25),I,J,DPSLUG(60),FREQH(60),FREQP(60),
     3 FREQN(60),VELH(60),VELP(60),VELN(60),PRESSH(60),
     4 PRESSP(60),PRESSN(60),K,KK
      REAL M,N
C
C     BLASIUS EQUATION NUMERATORS AND EXPONENTS
C
      COEFFL=16.0
      COEFFG=16.0
      N=1.0
      M=1.0
C
C     CALCULATION OF PARAMETERS X,RL,AND PHIL OF THE
C     LOCKHART-MARTINELLI CORRELATION
C
      X=SQRT((RATL(J)/RATG(I))*(DENSG/DENSL)*(VISCL/VISCG))
      X=ALOG10(X)
      PHIL=.4253-.4227*X+.2118*(X**2)-.3480E-01*(X**3)
     1 -.6754E-02*(X**4)
      RL=-.6431+.4818*X-.1289*(X**2)+.2323E-01*(X**3)
     1 -.2267E-02*(X**4)
C
C     ANTILOGS OF X, RL, AND PHIL
C
      X=10.0**X
      RL=10.0**RL
      PHIL=10.0**PHIL
      PHIG=X*PHIL
      RETURN
      END
```

```
      SUBROUTINE VISTUR
C
C     SUBROUTINE VISCOUS-TURBULENT
C     CALCULATION OF THE PARAMETERS OF THE LOCKHART-MARTIN-
C     ELLI CORRELATION FOR THE LIQUID IN VISCOUS FLOW AND
C     THE GAS IN TURBULENT FLOW
C
      COMMON VISCG,VISCL,DIA,PI,GEE,DENSL,DENSG,RELP,REGP,
     1 KORFUN,DPL,DPKOR ,COEFFL,COEFFG,PHIL,PHIG,RL,N,M,
     2 RATL(25),RATG(25),I,J,DPSLUG(60),FREQH(60),FREQP(60),
     3 FREQN(60),VELH(60),VELP(60),VELN(60),PRESSH(60),
     4 PRESSP(60),PRESSN(60),K,KK
      REAL M,N
C
C     BLASIUS EQUATION NUMERATORS AND EXPONENTS
C
      COEFFL=16.0
      COEFFG=0.046
      N=1.0
      M=0.2
C
C     CALCULATION OF PARAMETERS X,RL,AND PHIL OF THE
C     LOCKHART-MARTINELLI CORRELATION
C
      X=SQRT((COEFFL/COEFFG)*(REGP**(-0.8))*(RATL(J)/RATG(I))
     1 *(DENSG/DENSL)*(VISCL/VISCG))
      X=ALOG10(X)
      PHIL=.5396-.4661*X+.1531*(X**2)-.2207E-01*(X**3)
     1 +.2642E-03*(X**4)
      RL=-.6431+.4818*X-.1289*(X**2)+.2323E-01*(X**3)
     1 -.2267E-02*(X**4)
C
C     ANTILOGS OF X, RL, AND PHIL
C
      X=10.0**X
      RL=10.0**RL
      PHIL=10.0**PHIL
      PHIG=X*PHIL
      RETURN
      END
```

```
      SUBROUTINE TURVIS
C
C     SUBROUTINE TURBULENT-VISCOUS
C     CALCULATION OF THE PARAMETERS OF THE LOCKHART-MARTIN-
C     ELLI CORRELATION FOR THE LIQUID IN TURBULENT FLOW AND
C     THE GAS IN VISCOUS FLOW
C
      COMMON VISCG,VISCL,DIA,PI,GEE,DENSL,DENSG,RELP,REGP,
     1 KORFUN,DPL,DPKOR  ,COEFFL,COEFFG,PHIL,PHIG,RL,N,M,
     2 RATL(25),RATG(25),I,J,DPSLUG(60),FREQH(60),FREQP(60),
     3 FREQN(60),VELH(60),VELP(60),VELN(60),PRESSH(60),
     4 PRESSP(60),PRESSN(60),K,KK
      REAL M,N
C
C     BLASIUS EQUATION NUMERATORS AND EXPONENTS
C
      COEFFL=0.046
      COEFFG=16.0
      N=0.2
      M=1.0
C
C     CALCULATION OF PARAMETERS X,RL,AND PHIL OF THE
C     LOCKHART-MARTINELLI CORRELATION
C
      X=SQRT((COEFFL/COEFFG)*(RELP**0.8)*(RATL(J)/RATG(I))*
     1 (DENSG/DENSL)*(VISCL/VISCG))
      X=ALOG10(X)
      PHIL=.5433-.4406*X+.1443*(X**2)-.2778E-01*(X**3)
     1 +.1663E-02*(X**4)
      RL=-.6431+.4818*X-.1289*(X**2)+.2323E-01*(X**3)
     1 -.2267E-02*(X**4)
C
C     ANTILOGS OF X, RL, AND PHIL
C
      X=10.0**X
      RL=10.0**RL
      PHIL=10.0**PHIL
      PHIG=X*PHIL
      RETURN
      END
```

```
      SUBROUTINE TURTUR
C
C     SUBROUTINE TURBULENT-TURBULENT
C     CALCULATION OF THE PARAMETERS OF THE LOCKHART-MARTIN-
C     ELLI CORRELATION FOR BOTH THE LIQUID AND THE GAS IN
C     TURBULENT FLOW
C
      COMMON VISCG,VISCL,DIA,PI,GEE,DENSL,DENSG,RELP,REGP,
     1 KORFUN,DPL,DPKOR ,COEFFL,COEFFG,PHIL,PHIG,RL,N,M,
     2 RATL(25),RATG(25),I,J,DPSLUG(60),FREQH(60),FREQP(60),
     3 FREQN(60),VELH(60),VELP(60),VELN(60),PRESSH(60),
     4 PRESSP(60),PRESSN(60),K,KK
      REAL M,N
C
C     BLASIUS EQUATION NUMERATORS AND EXPONENTS
C
      COEFFL=0.046
      COEFFG=0.046
      N=0.2
      M=0.2
C
C     CALCULATION OF PARAMETERS X,RL,AND PHIL OF THE
C     LOCKHART-MARTINELLI CORRELATION
C
      X=SQRT(((RATL(J)/RATG(I))**1.8)*(DENSG/DENSL)
     1 *((VISCL/VISCG)**0.2))
      X=ALOG10(X)
      PHIL=.6275-.5013*X+.1308*(X**2)-.8409E-02*(X**3)
     1 -.2251E-02*(X**4)
      RL=-.6431+.4818*X-.1289*(X**2)+.2323E-01*(X**3)
     1 -.2267E-02*(X**4)
C
C     ANTILOGS OF X, RL, AND PHIL
C
      X=10.0**X
      RL=10.0**RL
      PHIL=10.0**PHIL
      PHIG=X*PHIL
      RETURN
      END
```

```
      SUBROUTINE KORDY
C
C     KORDYBAN CORRELATION
C
C
      COMMON VISCG,VISCL,DIA,PI,GEE,DENSL,DENSG,RELP,REGP,
     1 KORFUN,DPL,DPKOR ,COEFFL,COEFFG,PHIL,PHIG,RL,N,M,
     2 RATL(25),RATG(25),I,J,DPSLUG(60),FREQH(60),FREQP(60),
     3 FREQN(60),VELH(60),VELP(60),VELN(60),PRESSH(60),
     4 PRESSP(60),PRESSN(60),K,KK
      REAL M,N,KORFUN
C
C     CALCULATION OF THE KORDYBAN FUNCTION
C
      KORFUN=1.0+(DENSL/DENSG)*(RATG(I)/RATL(J))
C
C     THE SINGLE PHASE LIQUID PRESSURE DROP FROM THE
C     LOCKHART-MARTINELLI CORRELATION IS USED TO CALCULATE
C     THE PREDICTED PRESSURE DROP BY THE KORDYBAN CORRELATION
C
C     KORDYBAN TWO PHASE PRESSURE DROP
C
      DPKOR =DPL*(KORFUN**.75)
      RETURN
      END
```

```
      SUBROUTINE SLUG
C
C     CALCULATION OF THE PREDICTED PRESSURE GRADIENT BY THE
C     SLUG FLOW MODEL
C
      COMMON VISCG,VISCL,DIA,PI,GEE,DENSL,DENSG,RELP,REGP,
     1 KORFUN,DPL,DPKOR ,COEFFL,COEFFG,PHIL,PHIG,RL,N,M,
     2 RATL(25),RATG(25),I,J,DPSLUG(60),FREQH(60),FREQP(60),
     3 FREQN(60),VELH(60),VELP(60),VELN(60),PRESSH(60),
     4 PRESSP(60),PRESSN(60),K,KK
      DIMENSION RESLUG(60),FFSLUG(60),DPLSL(60),DPMOMH(60),
     1 QL(60),QG(60),V(60),FRACTL(60),NSLUG(60)
      REAL M,N
      REAL NSLUG
      IF(FREQH(K).EQ.0.0) GO TO 2
C
C     THE MASS RATES ARE CONVERTED TO VOLUMETRIC FLOW RATES
C     CUBIC FEET PER SECOND
C
      QL(J)=RATL(J)/(DENSL*3600.0)
      QG(I)=RATG(I)/(DENSG*3600.0)
C
C     THE FRACTION LIQUID IN THE PIPE IS CALCULATED
C
      FRACTL(K)=QL(J)/(QL(J)+QG(I))
C
C     OVERALL FLUID VELOCITY
C
      V(K)=4.0*(QL(J)+QG(I))/(PI*(DIA**2))
C
C     THE REYNOLDS NUMBER IS CALCULATED AND USED TO CALCULATE
C     THE FRICTION FACTOR BY THE BLASIUS EQUATION
C     (TURBULENT FLOW IS ASSUMED)
C
      RESLUG(K)=DIA*V(K)*DENSL/(VISCL*6.72E-04)
      FFSLUG(K)=0.0791/(RESLUG(K)**0.25)
C
C     PRESSURE DROP FROM LIQUID AT FLUID VELOCITY
C
      DPLSL(K)=2.0*FFSLUG(K)*DENSL*(V(K)**2)/(GEE*DIA*144.0)
      FREQH(K)=FREQH(K)/60.0
C
C     THE NUMBER OF SLUGS IN THE TEST SECTION IS CALCULATED
C
      NSLUG(K) =FREQH(K)/V(K)*18.29
C
C     CALCULATION OF THE EXPONENTIAL TERM FOR THE FILM
```

```
C      SLOWDOWN
C
       B=(PI**2)*(VISCL*6.72*10E-4/DENSL)/(FREQH(K)*4.0*(((RL
      1-FRACTL(K))*DIA)**2))
C
C      SUMMATION IN MOMENTUM TERM
C
       SUM1=0.0
       DO 1 II=1,5
       A=(2.0*FLOAT(II)-1.0)**2
       SUM1=SUM1+1.0/A*EXP(-A*B)
     1 CONTINUE
C
C      CALCULATION OF THE MOMENTUM LOSS OF THE FILM
C
       DPMOMH(K)=NSLUG(K) *DENSL*(RL-FRACTL(K))*(V(K)**2)/
      1 (GEE*144.0*18.29)*(1.0-64.0/(PI**4)*(SUM1**2))
C
C      TOTAL SLUG PRESSURE DROP PREDICTION
C
       DPSLUG(K)= DPLSL(K)*FRACTL(K)+DPMOMH(K)
       GO TO 3
     2 DPSLUG(K)=0.0
     3 RETURN
       END
```

TABLE B-1

CALCULATED DATA

RATES (LB/HR) WATER	AIR	PRESSURE GRADIENT (PSI/FT) L-M	KORDY	SLUG	HORIZ	POS	NEG
264.255	0.650	0.01413	0.01014	0.01884	0.01422	0.03280	0.0
358.988	0.650	0.02145	0.01515	0.02731	0.02078	0.04265	0.0
438.763	0.650	0.02833	0.01990	0.03461	0.02515	0.04483	0.00328
513.553	0.650	0.03535	0.02477	0.04082	0.03171	0.05796	0.00820
583.356	0.650	0.04235	0.02967	0.04704	0.03718	0.06561	0.01312
653.159	0.650	0.04977	0.03491	0.05392	0.04483	0.07326	0.01804
722.963	0.650	0.05759	0.04049	0.06020	0.05139	0.08201	0.02351
787.780	0.650	0.06520	0.04597	0.06584	0.05796	0.09295	0.03116
847.612	0.650	0.07251	0.05127	0.07119	0.06452	0.09841	0.03554
902.457	0.650	0.07944	0.05634	0.07601	0.07108	0.10388	0.04155
264.255	1.954	0.01911	0.01937	0.03802	0.02734	0.04374	0.0
358.988	1.954	0.02835	0.02767	0.05708	0.03937	0.05467	0.02515
438.763	1.954	0.03694	0.03514	0.07273	0.05249	0.06780	0.03280
513.553	1.954	0.04563	0.04253	0.08332	0.06124	0.08201	0.04702
583.356	1.954	0.05424	0.04976	0.09974	0.07217	0.09295	0.05577
653.159	1.954	0.06333	0.05730	0.11662	0.08748	0.10935	0.06452
722.963	1.954	0.07287	0.06515	0.13162	0.09295	0.12028	0.07654
787.780	1.954	0.08211	0.07271	0.14608	0.10935	0.12575	0.08748
847.612	1.954	0.09096	0.07992	0.15769	0.12028	0.14215	0.09841
902.457	1.954	0.09934	0.08673	0.16906	0.13122	0.14762	0.10935
264.255	3.228	0.03235	0.02711	0.05212	0.03062	0.06014	0.0
358.988	3.228	0.04547	0.03824	0.07852	0.05467	0.07654	0.04374
438.763	3.228	0.05729	0.04810	0.10043	0.07654	0.09295	0.07108
513.553	3.228	0.06898	0.05771	0.11805	0.09841	0.11482	0.08201
583.356	3.228	0.08039	0.06700	0.14118	0.11482	0.13122	0.10388
653.159	3.228	0.09226	0.07659	0.15839	0.12575	0.14215	0.12028
722.963	3.228	0.10458	0.08648	0.18128	0.14215	0.15309	0.13122
787.780	3.228	0.11641	0.09593	0.20293	0.15856	0.17496	0.14215
847.612	3.228	0.12765	0.10488	0.21846	0.16949	0.18043	0.15309
902.457	3.228	0.13822	0.11326	0.23848	0.17496	0.19683	0.16949

RATES (LB/HR)		PRESSURE GRADIENT (PSI/FT)					
WATER	AIR	L-M	KORDY	SLUG	HORIZ	POS	NEG
264.255	5.098	0.04562	0.03728	0.05565	0.04374	0.07654	0.0
358.988	5.098	0.06263	0.05220	0.10500	0.08201	0.09841	0.07108
438.763	5.098	0.07776	0.06523	0.12841	0.10388	0.12575	0.09295
513.553	5.098	0.09256	0.07783	0.16033	0.13669	0.15309	0.12028
583.356	5.098	0.10690	0.08991	0.19353	0.16402	0.17496	0.14215
653.159	5.098	0.12173	0.10228	0.21816	0.18589	0.21323	0.16402
722.963	5.098	0.13703	0.11495	0.24535	0.19683	0.22417	0.18043
787.780	5.098	0.15165	0.12697	0.27302	0.22963	0.22963	0.21323
847.612	5.098	0.16549	0.13828	0.29274	0.24057	0.25150	0.22963
902.457	5.098	0.17845	0.14883	0.33169	0.25697	0.26791	0.22963
264.255	7.731	0.06441	0.05022	0.0	0.0	0.0	0.0
358.988	7.731	0.08649	0.06998	0.10024	0.10935	0.12575	0.0
438.763	7.731	0.10585	0.08711	0.16068	0.15309	0.15309	0.13669
513.553	7.731	0.12462	0.10356	0.21507	0.18589	0.19136	0.18043
583.356	7.731	0.14268	0.11924	0.26174	0.22417	0.23510	0.19683
653.159	7.731	0.16124	0.13522	0.29800	0.24604	0.26244	0.23510
722.963	7.731	0.18029	0.15149	0.33545	0.27884	0.28431	0.26244
787.780	7.731	0.19840	0.16685	0.37154	0.30618	0.31165	0.29524
847.612	7.731	0.21548	0.18124	0.39803	0.32258	0.0	0.31165
902.457	7.731	0.23143	0.19462	0.42652	0.32805	0.0	0.32805
264.255	11.172	0.08970	0.06562	0.0	0.0	0.0	0.0
358.988	11.172	0.11799	0.09116	0.0	0.0	0.0	0.0
438.763	11.172	0.14252	0.11320	0.0	0.0	0.20776	0.0
513.553	11.172	0.16612	0.13427	0.23811	0.22963	0.25150	0.21870
583.356	11.172	0.18866	0.15428	0.31101	0.27337	0.28431	0.25697
653.159	11.172	0.21170	0.17459	0.38175	0.31711	0.32258	0.30618
722.963	11.172	0.23523	0.19520	0.42568	0.34992	0.35539	0.32258
787.780	11.172	0.25750	0.21460	0.0	0.0	0.0	0.37179
847.612	11.172	0.27843	0.23273	0.0	0.0	0.0	0.0
902.457	11.172	0.29791	0.24953	0.0	0.0	0.0	0.0

APPENDIX - C

SAMPLE CALCULATIONS

(1) LOCKHART-MARTINELLI CORRELATION

(2) KORDYBAN CORRELATION

(3) SLUG FLOW MODEL

PROBLEM: To predict the two-phase pressure drop in a 0.5 inch diameter tube.

Water Rate = 1.31 USGPM

Air Rate = 0.76 CFM

PHYSICAL PROPERTIES

(AT 80°F and 1 ATM)

	VISCOSITY (CENTIPOISE)	DENSITY (LB/FT^3)
Air	0.018	0.0708
Water	0.900	62.20

1. Solution by Lockhart-Martinelli Correlation

Convert rates to lb/hr

$W_L = Q_L\ (60.0)(.1336)\ \rho_L$

$= (1.31)(60.0)(.1336)(62.2)$

$= 653.16$ lb/hr

$W_G = Q_G\ (60.0)\ \rho_G$

$= (0.76)(60.0)(0.0708)$

$= 3.228$ lb/hr

Calculation of the gas and liquid Reynold's Numbers if each is assumed to flow alone.

$Re_L = (4.0)\ W_L/\mu(2.4191)\pi D$

$= (4.0)(653.16)/(0.900)(2.4191)\pi(0.0417)$

$= 9160.0$

$Re_G = (4.0)\ W_G/\mu(2.4191)\pi D$

$= (4.0)(3.228)/(0.018)(2.4191)\pi(.0417)$

$= 2263.8$

Since the Reynolds numbers are above 2000, both phases are in turbulent flow.

The Blasius numerators and exponents for this case are:

liquid	gas
$n = 0.2$	$m = 1.0$
$C_L = 0.046$	$C_G = 0.046$

$$f_L = 0.046/(Re_L)^{2.0}$$

$$= 0.046/(9160.0)^2$$

$$= 0.0074$$

Pressure drop in the liquid phase

$$\Delta P_L = (32.0)\ f_L W_L{}^2/\rho_L D^5 \pi^2 g_c\ (3600.0)^2(144.0)$$

$$= (32.0)(.0074)(653.16)^2/(62.2)(0.0417)^5\pi^2(32.17)\ (3600.0)^2(144.0)$$

$$= 0.02180 \text{ psi/ft}$$

Calculation of Parameter $X = \left(\dfrac{(\frac{\Delta P}{\Delta L})_L}{(\frac{\Delta P}{\Delta L})_G}\right)^{0.5}$

for turbulent-turbulent flow

$$X^2 = \left(\frac{W_L}{W_G}\right)^{1.8}\left(\frac{\rho_G}{\rho_L}\right)\left(\frac{\mu_L}{\mu_G}\right)^{0.2}$$

$$= \left(\frac{653.16}{3.228}\right)^{1.8}\left(\frac{0.0708}{62.2}\right)\left(\frac{0.900}{0.018}\right)^{0.2}$$

$$X^2 = 35.3$$

$$X = 5.94$$

from Lockhart-Martinelli's Coordinates of Φ and R_L versus parameter X (curve-fit in the Computer Program in Appendix B)

$$\Phi_L = \left(\frac{(\frac{\Delta P}{\Delta L})_{TP}}{(\frac{\Delta P}{\Delta L})_L}\right)^{0.5} = 2.057$$

R_L = liquid holdup = 0.4596

Therefore

$$\Delta P_{TP} = \Phi_L{}^2 \ \Delta P_L$$

$$= (2.057)^2 \ (0.02180)$$

$$= 0.09226 \text{ psi/ft}$$

Pressure Drop Prediction by Lockhart-Martinelli = 0.09226 psi/ft.

2. Kordyban Correlation

Calculation of the Kordyban function

$$\text{KORFUN} = 1.0 + (\rho_L/\rho_g)(W_G/W_L)$$

$$= 1.0 + (62.2/0.0708)(3.228/653.16)$$

$$= 5.342$$

Calculation of single phase (liquid) pressure drop. Kordyban assumed only turbulent flow in smooth tubes having a Reynold's number of less than 100,000.

Therefore $\text{Re} = \dfrac{0.079}{\text{Re}^{0.25}}$

To extend the range of Kordyban's correlation, the Blasius equation used by Lockart-Martinelli was assumed to be more applicable.

Therefore, ΔP from Lockhart-Martinelli is

$$\Delta P_L = 0.02180 \text{ psi/ft.}$$

Therefore

$$\Delta P_{TP} = \Delta P_L \ (\text{KORFUN})^{0.75}$$

$$= 0.02180 \ (5.342)^{0.75}$$

$$= 0.07659 \text{ psi/ft}$$

Pressure Drop Prediction by Kordyban's Correlation

= 0.07659 psi/ft

3. Slug Model Theory

GIVEN: Slug frequency = 120/Min

Calculation of the fraction liquid in the total flow.

$$FRACTL = Q_L/(Q_L + Q_G)$$

$$= \left(\frac{10.50}{10.50 + 45.6}\right)$$

$$= 0.187$$

Calculation of fluid velocity

$$u = (Q_L + Q_G)/\frac{\pi D^2}{4}$$

$$= (56.10)(4.0)/\pi(.0417)^2(3600)$$

$$= 11.4 \text{ ft/sec}$$

Calculation of frictional pressure drop term.

Fluid Reynolds number, assuming fluid density to be that of the liquid.

$$Re_s = Du\rho_L/\mu$$

$$= (0.0417)(11.4)(62.2)/(0.900)(6.72 \times 10^{-4})$$

$$= 48935.1$$

Friction factor by Blasius equation

$$f_s = 0.0791/Re^{0.25}$$

$$= 0.0791/(48935.1)^{0.25}$$

$$= 0.00532$$

Frictional pressure drop of the slugs

$$\Delta P_s = ((2.0)f\rho_L\ u^2/g_c D(144.0))\left(\frac{Q_L}{Q_L + Q_G}\right)$$

$$= ((2.0)(0.00532)(62.2)(11.4)^2/(32.17)(0.0417)(144.0))(0.187)$$

$$= 0.0834 \text{ psi/ft}$$

Calculation of momentum loss term. Number of slugs in the test section

$$N_s = \frac{\text{slug frequency}}{u} L_t$$

$$= (\frac{2.0}{11.4})(18.29)$$

$$= 3.20 \text{ slugs/section}$$

Calculation of the exponential term for film slowdown

$$B = (\pi^2 \mu_L (6.72 \times 10^{-4})/\rho_L)/((\text{Slug frequency})(4.0)(R_L - \text{FRACTL})D^2)$$

$$= (\pi^2(0.900)(6.72 \times 10^{-4})/(62.4))/((2.0)(4.0)(.4596 - .187)(0.0417)^2)$$

$$= 0.9296$$

Calculation of summation in the momentum term

$$SUM = \sum_1^{\infty} (1.0/(2n-1)^2)e^{-(2n-1)^2 B}$$

Since this term converges very quickly, the summation was taken from 1 to 5, only

$$SUM = 0.3947$$

Therefore,

Total momentum loss term

$$\Delta P_m = (N_s \rho_L (R_L - \text{FRACTL})u^2/g_c(144.0)L_t)((1.0 - (64.0/\pi^4))(SUM)^2)$$

$$= ((3.20)(62.2)(0.4596 - 0.187)(11.4)^2/(32.17)(144.0)(18.29))((1.0 - (64.0/\pi^4))(0.3947)^2)$$

$$= 0.07493 \text{ psi/ft}$$

Therefore the total pressure drop by the slug flow model is

$$\Delta P_{TOT} = \Delta P_s + \Delta P_m$$

$$= 0.0834 + 0.0749$$

$$= 0.1583 \text{ psi/ft}$$

Actual Pressure drop was 0.1258 psi/ft

Comparison

Relationship	ΔP_{TOT}	Deviation from actual
Lockhart-Martinelli	0.09226	-26.6%
Kordyban	0.07659	-39.1%
Slug Model	0.1583	+25.8%
Actual	0.1258	--

APPENDIX - D

INSTRUMENT CALIBRATION

(1) ROTAMETERS

(2) PRESSURE TRANSDUCERS

ROTAMETER CALIBRATION

Factory calibration curves were supplied with each of the Brooks Rotameters

AIR	WATER
#6709-31235	#6709-31236
#6709-2288-1	

No curves were available for #624-675-1.

All of the meters were recalibrated for the respective fluids at 80°F.

Calibration of the air Rotameters was carried out with the use of a VOL-O-FLO area meter (National Instrument Laboratories, Inc.) (Mod. 20 ER) which has a range of zero to 300 scfh. This meter was previously calibrated with a volume displacement meter.

The results are shown in Table D-1 and are plotted in Figures 28 and 29.

Calibration of the water Rotameters was carried out by measuring the flow into a graduated cylinder over specific time intervals. The flow rates, in milliliters per minute were converted to U.S. gallons per minute versus the scale readings. The resultant data are shown in Table D-2 and are plotted in Figures 30 and 31.

TABLE D-1

ROTAMETER

CALIBRATION DATA
WITH WATER SATURATED AIR AT 80°F

Brooks Rotameter #6709-2288-1

Model 8-1100, Tube No. R-8M-25-4, Float 8RV3

Scale Reading	cfm (80°F, 14.65 psia)
10	.392
20	.650
30	.917
40	1.20
50	1.475
60	1.82
70	2.18
80	2.635
90	3.15
100	3.70

Brooks Rotameter #6709-31235

Model 1110 Tube R-6L-600-G1, Float 316SS (RS Spool)

Scale Reading	cfm (80°F, 14.65 psia)
10	.1525
20	.249
30	.317
40	.385
50	.460
60	.530
70	.600
80	.670
90	.765
100	.867

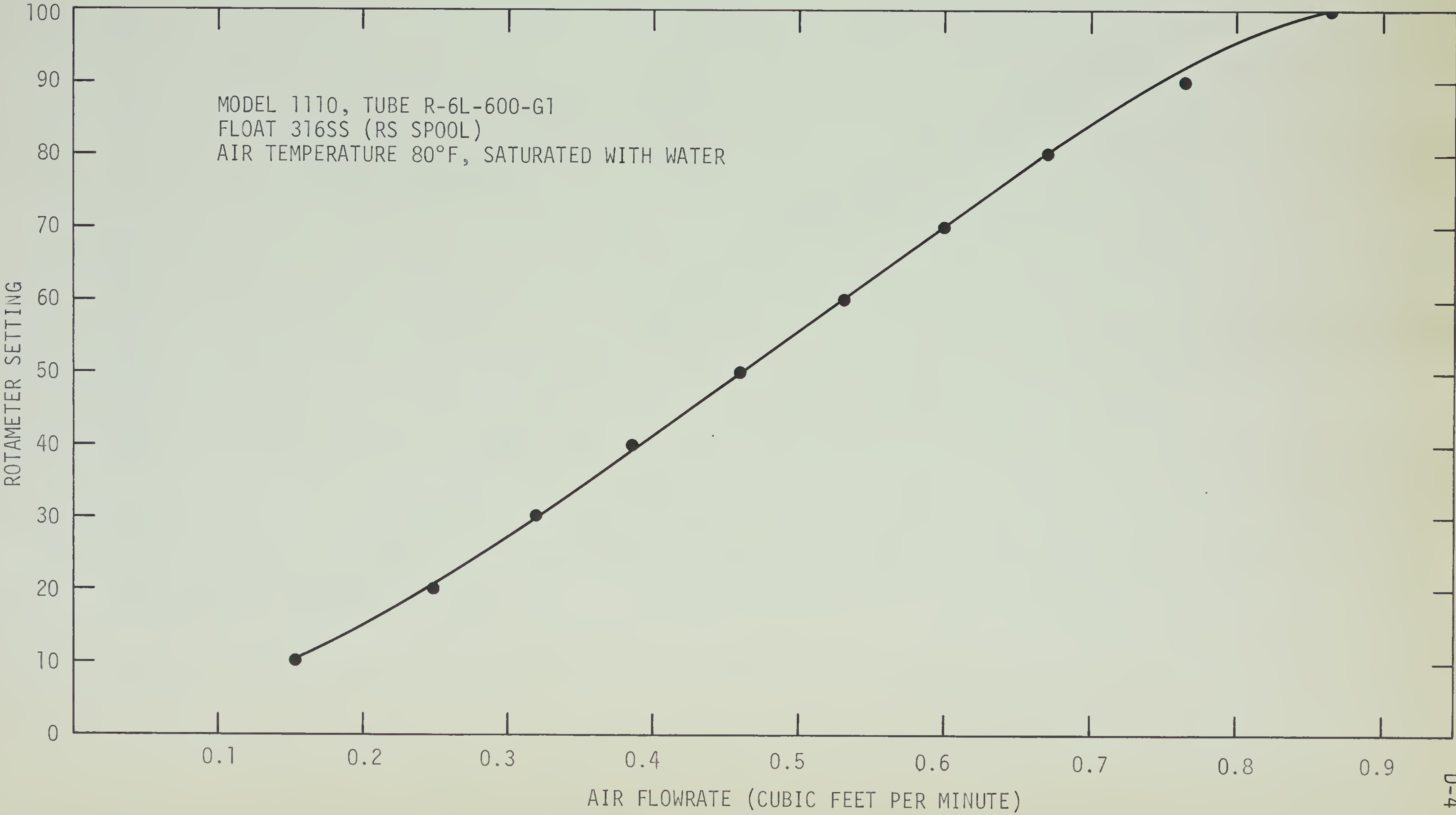


FIG. 28 - CALIBRATION OF BROOKS ROTAMETER #6709-31235

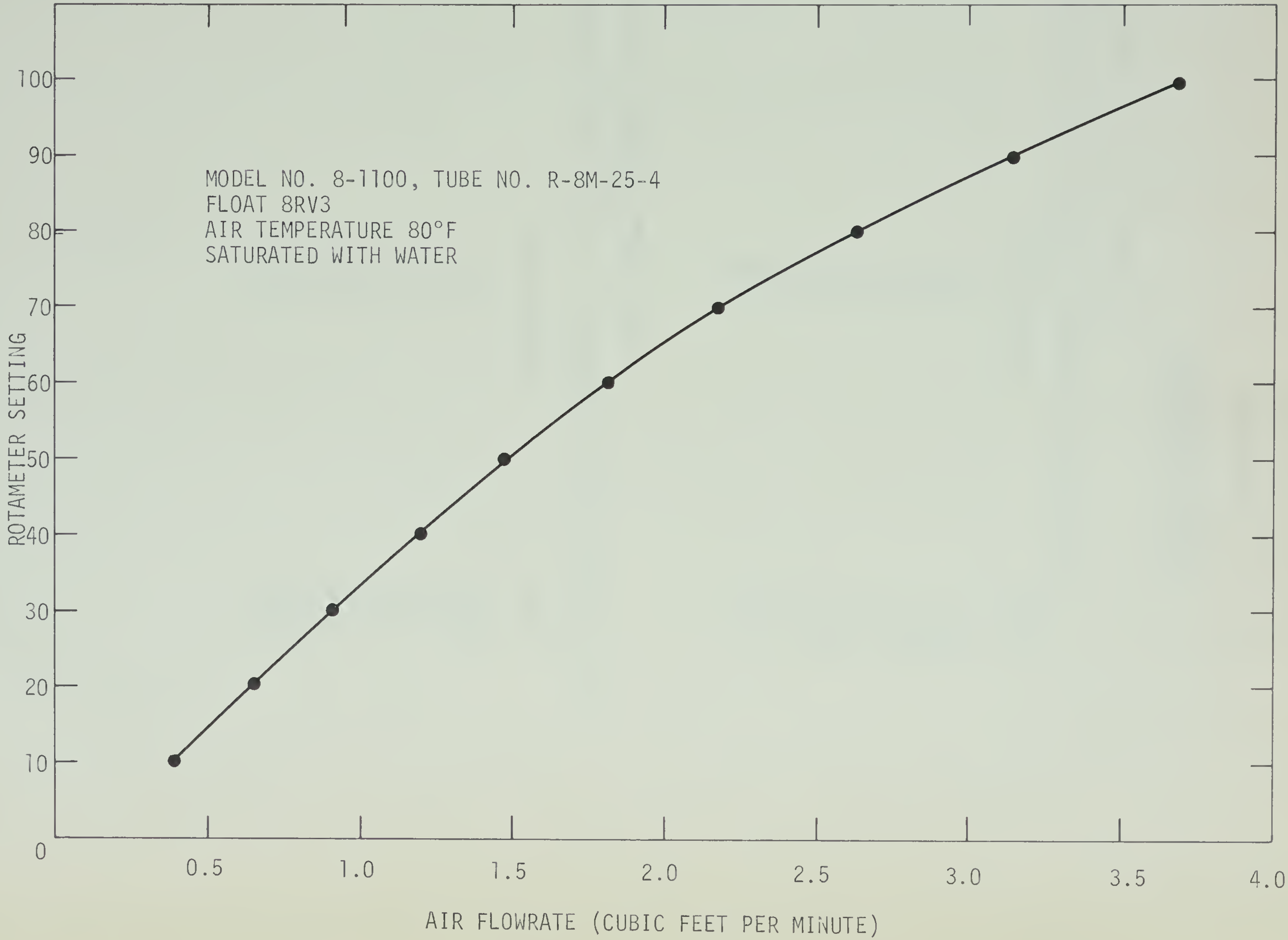


FIG. 29 - CALIBRATION OF BROOKS ROTAMETER #6709-2288-1

TABLE D-2

ROTAMETER

CALIBRATION DATA
WITH WATER AT 80°F

Brooks Rotameter #624-675-1

Model 8-1100, Tube No. R-8M-25-4, Float 8-RS-14

Scale Reading	USGPM
10	.526
20	.722
30	.887
40	1.024
50	1.16
60	1.305
70	1.445
80	1.58
90	1.70
100	1.80

Brooks Rotameter #6709-31236

Model 8-1110, Tube No. R-6L-600-Gl, Float 316SS(Rs Spool)

Scale Reading	USGPM
10	.0395
20	.0534
30	.077
40	.099
50	.116
60	.133
70	.152
80	.1695
90	.1915
100	.212

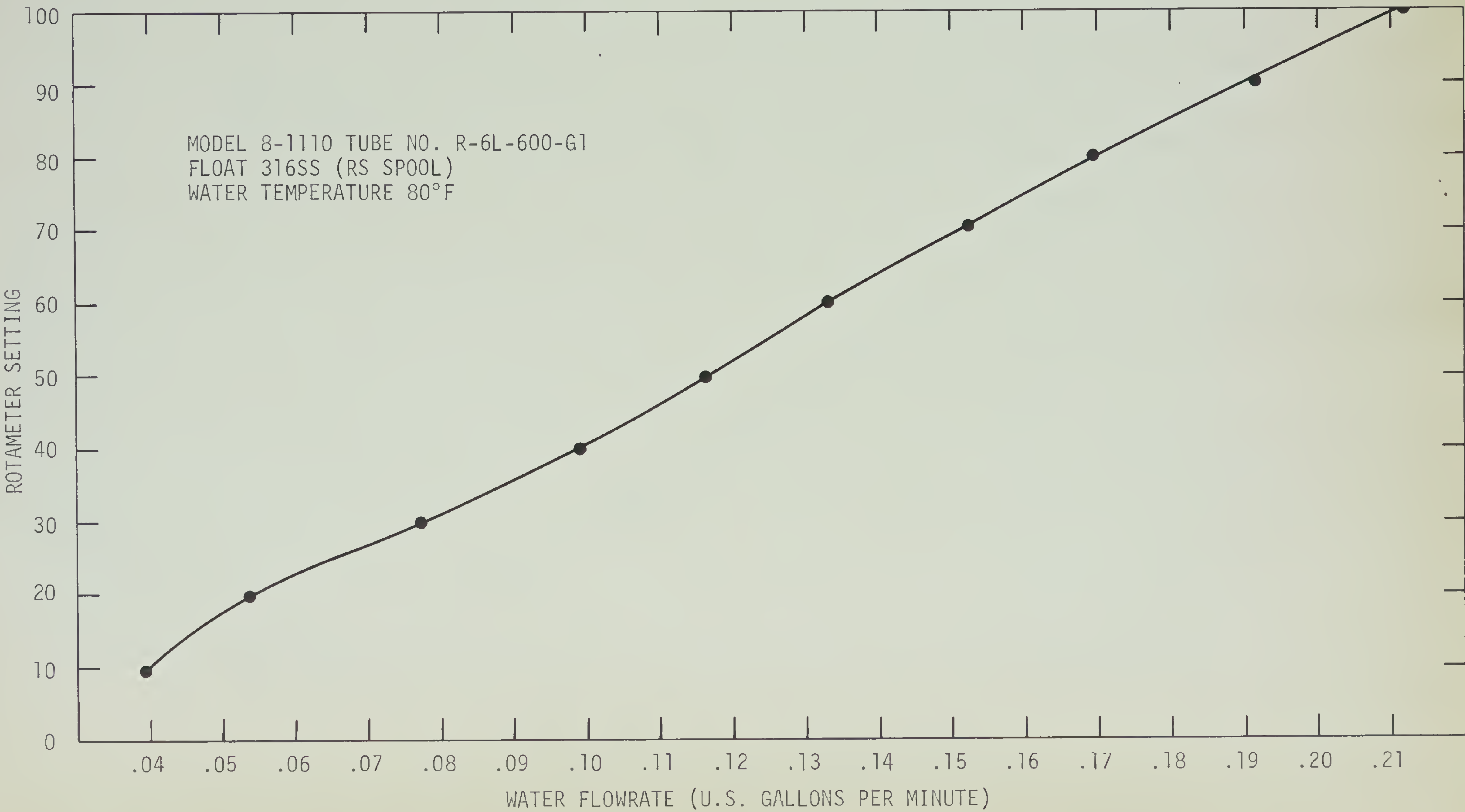


FIG. 30 - CALIBRATION OF BROOKS ROTAMETER #6709-31236

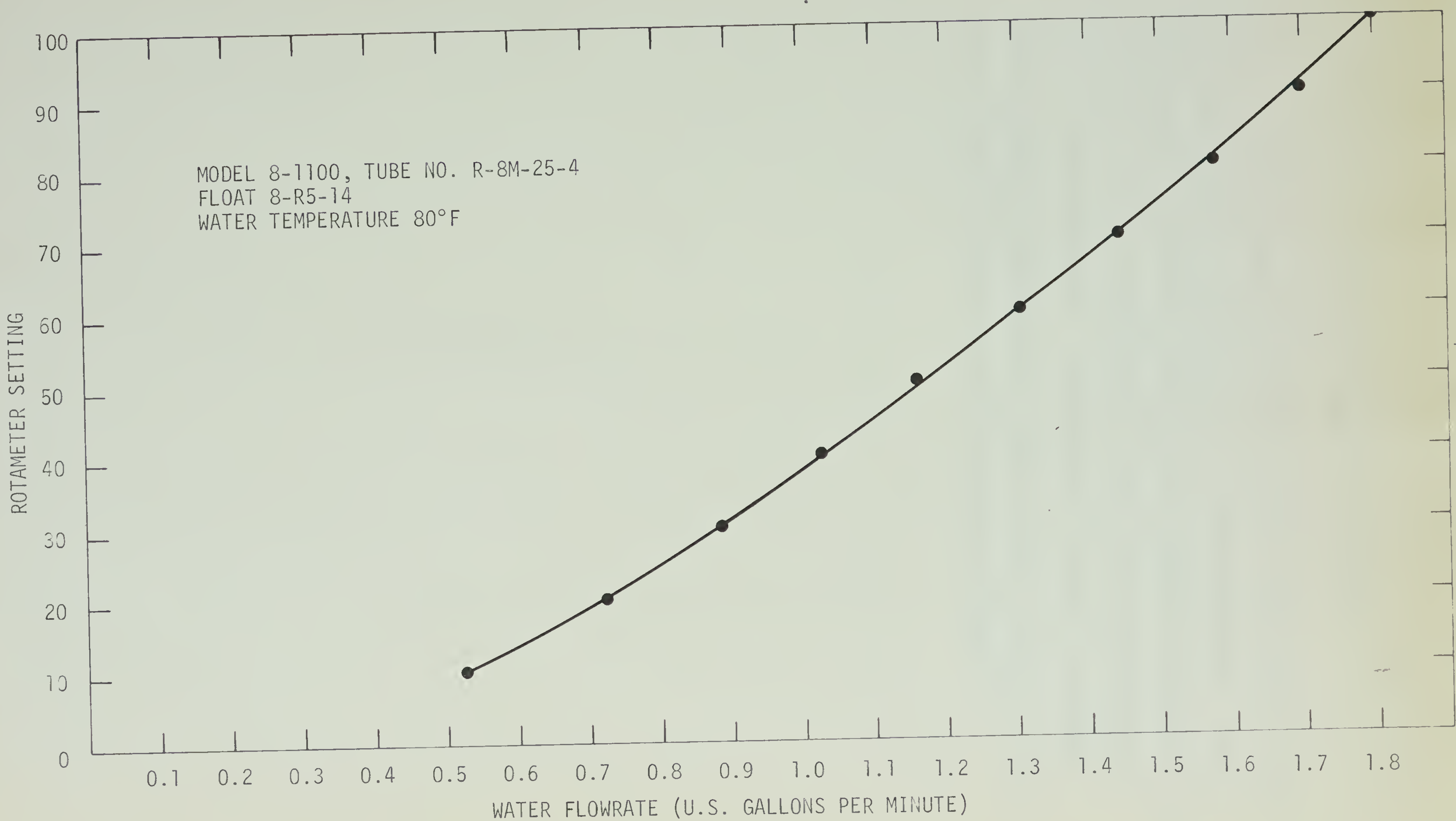


FIG. 31 - CALIBRATION OF BROOKS ROTAMETER #624-675-1

PRESSURE TRANSDUCER CALIBRATION

The two Statham Pressure Transducers Model 80TC ±15 - 350 #10153 and #3830 were calibrated in the following manner. Pressure was applied to the positive side, with the negative side open to the atmosphere, from the compressed air system and was measured with a mercury U-tube manometer. The output was recorded on a Hewlett-Packard (Moseley Autograf Model 7100B) stripchart recorder. The results are shown in Table D-3 and are plotted in Figure 32.

TABLE D-3

CALIBRATION DATA

FOR

STATHAM PRESSURE TRANSDUCERS

MODEL 80 TC ±15 - 350

#10153 AND #3830

Hewlett-Packard Carrier Preamp Setting: Atten = 20

Hewlett-Packard Autograf Recorder Setting: 5V

Autograf Reading	Pressure (PSIG)
1.0	1.03
2.0	2.06
3.0	3.10
4.0	4.12
5.0	5.15
6.0	6.19
7.0	7.20
8.0	8.24
9.0	9.27

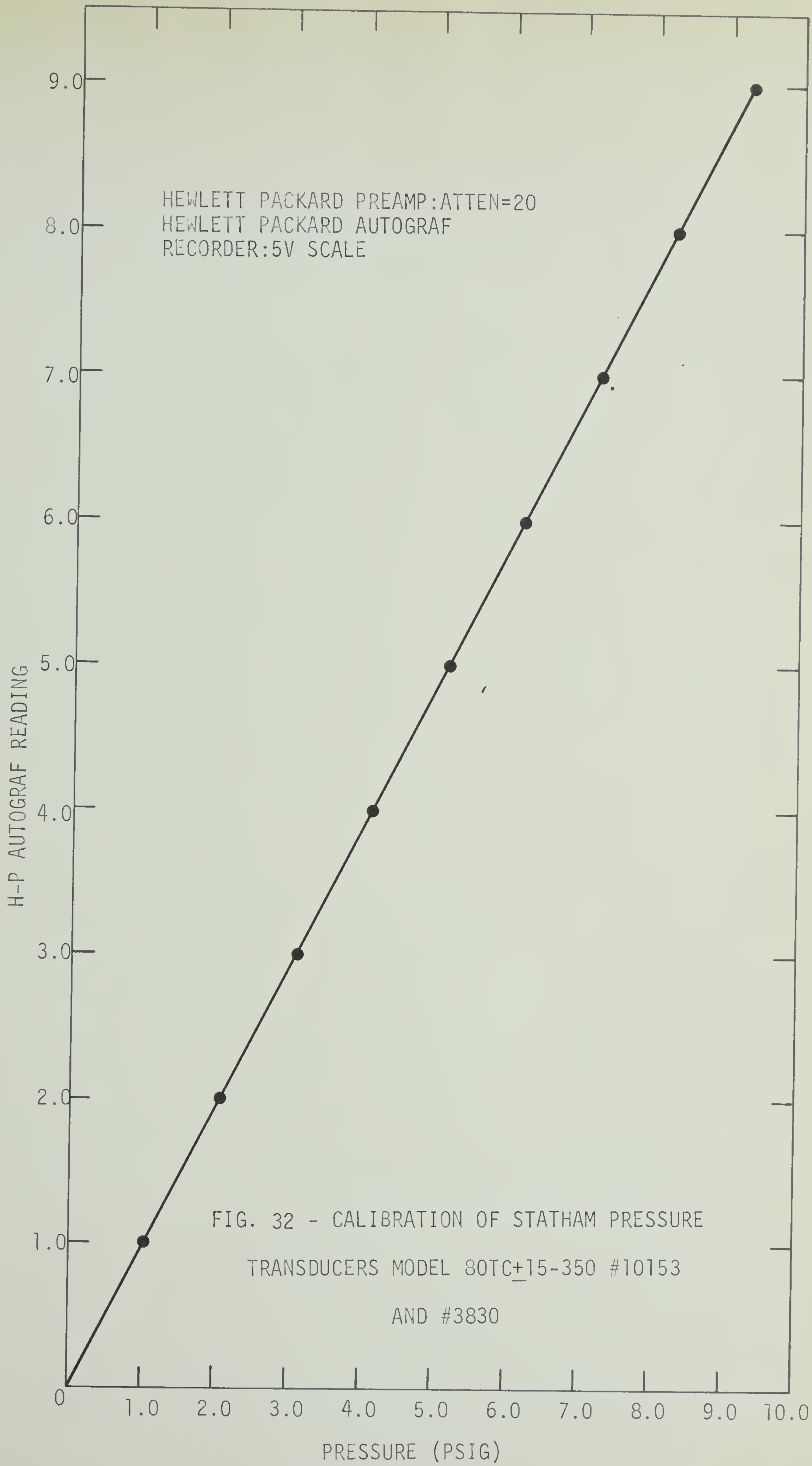


FIG. 32 - CALIBRATION OF STATHAM PRESSURE TRANSDUCERS MODEL 80TC±15-350 #10153 AND #3830